AN ECONOMIC ANALYSIS OF OPTIMUM POPULATION SIZE ACHIEVED THROUGH BOOSTING TOTAL FERTILITY AND NET IMMIGRATION

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Abstract

Without net immigration, Singapore's population size is projected to decline from 2025 onwards. Does it matter? To answer this question, the paper proceeds in two main parts. In the first part, taking a citizen's utility as a measure of welfare, we identify the channels through which a larger population size reduces welfare on the one hand, and increases welfare on the other. The optimum population size is achieved when the net resultant effect of all these channels leaves citizens' welfare at the maximum. When current and projected total fertility rates, without net immigration, lead to a projected path of actual population size that glides below the path of optimum population size, the policy question is how best to boost population increase to reach the optimum. The second part of the paper analyses the costs to Singapore society of reaching the optimum by measures to boost total fertility rate, on the one hand, and allowing net immigration flows on the other hand.

A starting point of economic analysis uses neoclassical growth theory to demonstrate how an increase in population size reduces per capita consumption and hence utility via a "capital dilution" channel. With a limited land size, an increase in population size raises population density, which lowers welfare through a "congestion" channel. The paper, however, identifies four other channels through which a larger population size increases welfare. These are a "tax base" channel, a "Mozart effect" channel, a "human capital externalities" channel, and an "Okun's Law" channel.

To analyse the costs to Singapore's national budget of boosting the total fertility rate, we start off with the classic Becker model of fertility decisions and quantity-quality trade-off. When parents value both the quantity as well as human capital level ("quality") of children, the Becker model predicts that when parents' incomes rise, they choose quality over quantity. (This can be called a "level" effect.) When education boosts a child's human capital, and higher growth rates raise the marginal productivity of parental investment in a child's human capital, the expected decline in GDP growth rates as the Singapore economy matures would boost total fertility. (This can be called a "growth" effect.) The impact of policy measures such as parental leave, childcare subsidy and the Baby Bonus on total fertility rate can be analysed in terms of substitution and income effects.

The costs of net immigration to Singapore society, apart from fiscal subsidies to attract potential immigrants, would appear to come from its impact on social capital. In particular, a recent concept of "identity economics" — that an individual's payoff or utility is affected by identification with particular social categories — can help us understand the nature of the cost of achieving a given increase in population via net immigration. The optimal mix of measures to boost total fertility rate and allowing net immigration flows to achieve a given increase in the size of population equates the marginal cost of the two approaches. Forging a national identity is an investment that can lower the cost of achieving a given increase in population size.

An Economic Analysis of Optimum Population Size Achieved Through Boosting Total Fertility and Net Immigration

Introduction

On current projections, if Singapore's total fertility rate remains at 1.2 with no net immigration into the economy, the citizen population size will start to shrink in 2025 (National Population and Talent Division 2012). In this scenario, the population size will be a little above 2.5 million by 2060.[1] To dramatise somewhat, let us ask what would happen to citizens' lives if the shrinkage persisted, total fertility rate stays below the replacement rate of 2.1, and we revert to the population size prevailing in 1970.[2] Will this hurt our future standard of living, jobs and wages, returning us back to life as it was in 1970? Does our economic prosperity depend on our population size?

Neoclassical growth theory provides us a first-pass answer to the above questions.[3] Central to the theory is the notion of an aggregate production function that relates an economy's output — total real GDP — to the economy's inputs. These inputs include labour, physical capital, human capital, and technology.[4] A common metric used to measure the standard of living is the real GDP per capita, that is, total real GDP divided by total population. Given the labour force participation rate, the standard of living is positively related to the amount of physical capital per worker ("machines per worker"), the amount of human capital per worker, and the state of technology adopted widely across the economy. In neoclassical growth theory, if in T years from today, our population size is down to two million, the level that prevailed in 1970, it does not follow that our standard of living in the year $2012 + T$ will revert to the 1970 level. This is because over the past 40 years, we have closed the technology gap between Singapore and the world's technology leaders, our huge investment in education over the years has raised the amount of human capital per worker, and the amount of physical capital per worker is unlikely to fall below the level that prevailed in 1970. For all these reasons, as a first-pass at the answer, neoclassical growth theory does not predict that the average Singaporean's standard of living will start to decline once citizen population size starts to shrink from 2025. In fact, there is a "capital dilution" effect where a high population growth rate leads to too many workers competing to use a given stock of machines such that average labour productivity declines on that account, thus lowering the standard of living.

Moreover, when we recognise land as a factor of production, there are limits to the growth of standard of living as the amount of usable land available for

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1. The citizen population size in 2011 is about 3.3 million (Yearbook of Statistics Singapore 2012, Table 3.1). http://www.singstat.gov.sg/pubn/reference/yos12/statsT-population.pdf.

2. According to Population Census 1970, the resident population size was a little above two million in 1970. http://www.singstat.gov.sg/stats/themes/people/hist/popn.html.

3. The classic paper introducing neoclassical growth theory is Solow (1956). A follow-up paper, Solow (1957), introduces growth accounting.

4. Mathematically, we express the relationship between aggregate output and inputs as $Y_t = F(K_t, h_t L_t, A_t)$, where Y_t is aggregate output or total real GDP, K_t is the total stock of physical capital such as machinery and equipment and structures, h_t is the stock of human capital per worker which refers to the average skill level embodied in labour that has been acquired through formal education as well as on-the-job training, L_t is the size of the labour force, and A_t represents the state of technology adopted by businesses in the economy.

production activity limits the number of workers that can work on the land without facing severe declines in labour productivity. As a first approximation, beyond some point, the rate of growth of usable land is zero. Then, a negative net growth of population starting from 2025 would actually lead to rising standards of living as land-labor intensity increases and the economy finds relief from the Law of Diminishing Marginal Productivity.[5] Historically, the Malthusian idea that a population size that is roughly proportional to land area, such that labour productivity sustains a standard of living at the subsistence level, has been found to have good predictive power at least until the Industrial Revolution (Kremer 1993). Since the Industrial Revolution, however, many countries have experienced a steady rise in the standard of living that has been fueled by technological progress, which enabled them to overcome the constraints of limited land. Nevertheless, for land-scarce Singapore, there are probably limits to how much technological progress can sustain a positive net population growth rate without causing severe overcrowding. A "congestion" channel places limits on population density and thus limits population growth.

Why then should we worry about a projected decline in citizen population size from 2025 at a current fertility rate of 1.2 with zero net immigration? The most compelling reason appears to come from recognising that, in any society, more than one generation of people co-exist. In the toolkit of an economist is an overlapping-generations model introduced by the late economist Paul Samuelson.[6] In such a model economy, there co-exist both young and old people. The young work and earn a wage income out of which they save for their old age when they are retired. At any one time, however, there are also the current retired old who live alongside the working young. In an unfunded pay-as-you-go (PAYG) social security system such as the one that prevails in the United States and many Western European economies, the young pay social security taxes that fund pension payments to the retired old. When the total population size declines, there are fewer young people who pay taxes to fund the pension liabilities per retired old person. This fiscal burden is increased with an increase in longevity unless the number of working years is correspondingly increased. Although Singapore's social security system, the Central Provident Fund, is not PAYG but a defined contribution system that includes a personal Medisave account, the public generally accepts a system of medical subsidies funded by the state. When the citizen old-age support ratio — that is, the ratio of persons aged 20 to 64 years to persons aged 65 and over — is projected to decline from a current level of 6.3 to less than 2 in 2060, this shrinkage of the "tax base" channel can be hugely important (NPTD 2012, Chart 8). Taking account of this channel can significantly raise the optimum population size.

There are three other reasons that make an increased population size desirable. Compared to 1970, when Singapore was largely a technology follower that could pick the low-hanging fruit of available technology through its openness to international trade and inflows of multinational corporations,

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5. As each worker has more land to work with, his or her marginal productivity rises.
6. See Samuelson (1958). Diamond (1965) is another classic reference, which introduces production into the Samuelson model.

we now have a mature economy that needs to generate its own innovations to maintain a high standard of living. If we suppose that innovation requires creative and talented people, and that the probability of being a creative and talented person is independently distributed among people, then it follows that we will find a greater number of creative and talented people within a larger population. This has been termed a "Mozart effect" by the economist Edmund Phelps, who introduced to economists one of his most memorable sentences, "If I could re-do the history of the world, halving population size each year from the beginning of time on some random basis, I would not do it for fear of losing Mozart in the process" (Phelps 1968, 512).

A related, though distinct, reason for desiring a larger population size has to do with the existence of positive human capital externalities. The notion here is that a person's return to his or her own investment in human capital is boosted by being able to draw upon a bigger pool of ideas that are embodied in a larger number of people. This leads to an agglomeration effect as highly skilled people benefit from being physically close together, often in cities. This positive "human capital externalities" channel offers another reason for desiring a larger population size.

Finally, there exists a strong positive historical relationship between Singapore's total real GDP growth rate (above a minimum level) and a change in the unemployment rate dubbed "Okun's Law" after the late economist Arthur Okun who first discovered this relationship in US data. Regressing the change in the unemployment rate on the rate of real GDP growth for Singapore over the period 1967–2002, the present author found that, historically, a one-percentage point shortfall of the real GDP growth rate below 7.1 percent in any given year results in a rise in the unemployment rate of 0.12 percentage points over the previous year (Hoon 2006, 64–8). The reputation that Singapore has built as a good investment destination allows it to grow its labour supply (by boosting population size) without a corresponding decline in labour productivity because capital grows to support the additional headcounts. The higher GDP growth, in turn, lowers the unemployment rate based on Okun's Law.[7]

If we are willing to make the judgment that the resultant net effect of the "capital dilution" channel, the "congestion" channel, the "tax base" channel, the "Mozart effect" channel, the "human capital externalities" channel, and the "Okun's Law" channel is to leave an optimum path of citizen population size that lies uniformly above the projected population path under the scenario of a total fertility rate of 1.2 and no net immigration, the next question is: how best to achieve the needed population size increase — through boosting total fertility rate or allowing net immigration flows? The second part of the paper addresses this question.

If we are to assess the effectiveness of policy measures to boost the total fertility rate, it is necessary that we have a theory of fertility decisions. The

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7. Hoon (2006) further notes that as workers and employers adjust to lower growth expectations (with a mature economy), the minimum real GDP growth rate required to maintain constant unemployment rate, calculated to be 7.1 percent based upon data for 1967 to 2002, is likely to decline.

pioneering paper in this area is Becker (1960) entitled, "An Economic Analysis of Fertility," where children are viewed as providing "utility" to parents. In a follow-up paper by Becker and Lewis (1973) entitled, "On the Interaction between the Quantity and Quality of Children," an analysis of how parents make a choice when faced with a trade-off between the number of children to have (the "quantity" choice) and the amount of human capital to provide each child with (the "quality" choice) is provided. [8] It is a widely observed phenomenon around the world that as nations launch onto a path of modern economic growth, they undergo a demographic transition where the total fertility rate declines. As individuals become richer, earning a higher market wage per unit of time spent working, the opportunity cost of the time devoted to raising a child correspondingly increases. In a household where traditionally the wife devotes more time to child-care, the substitution effect of a higher relative price of having another child tends to dominate the income effect. For a given budget for children's education, having fewer children also means that more can be spent on raising the human capital level of each child. Since achieving a higher level of human capital for each child also makes it more expensive to raise an additional child, there is a reinforcing effect making parents choose to have even fewer children. The slew of policy measures to boost total fertility rate, including parental leave, childcare subsidy and the Baby Bonus work within this framework via substitution and income effects.

Becoming richer leads parents to have fewer children who have higher levels of human capital. We may call this a "level" effect that can be distinguished from another effect that we may call a "growth" effect. The latter works as follows: more resources, including time, devoted to educating a child will raise the child's level of human capital. In an era of high growth rates, the potential to gain from parental investment in a child's education might be higher as higher skills are more valued in a rapidly growing economy. If this is true, then the implied growth slowdown as the Singapore economy matures could mean that total fertility rate would increase on this account. Whether the "growth" effect would dominate the "level" effect, thus reversing the trend toward reduced total fertility rate, is an empirical question.

While the cost of policy measures to boost the total fertility rate can be explicitly accounted for in the national budget[9], the cost to society of achieving a given increase in population size via net immigration appears to be more indirect and works through its effect on social capital. Suppose that we desire to have one more person added to the citizen population pool. We have a choice between two persons with the same skill levels; one has grown up as a citizen while the other is a potential immigrant. What is the essential difference that will affect our choice? Based on conventional economic analysis, there would not appear to be any essential difference. However, many people might say that it does make a difference and that the choice has important economic consequences. To see how this is so, it is very helpful to

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8. It is assumed that parents gain utility from having more children as well as from seeing their children have higher levels of human capital. ("Seeing my children graduate from the university gives me joy!")
9. The Marriage and Parenthood Package covering measures to help with child-raising costs, childcare support and time spent with young children, was first introduced in 2001 and cost the government S$1.6 billion a year starting from 2008 (NPTD 2012, 4).

draw insights from the recent work of economists George Akerlof and Rachel Kranton who introduced the concept of "identity economics".[10]

Akerlof and Kranton (2000) made a particular person's utility depend upon three arguments: his or her own actions, the actions of others, and the particular person's identity. By the particular person's identity, we mean that the person identifies with social categories that affect the marginal gain in utility from his or her own action; additionally, the actions of other people can cause this particular person to lose utility by affecting his or her identity. For example, Akerlof and Kranton cite evidence that the incentive pay (which is a wage premium over and above the level that would prevail in a perfectly competitive labour market) given to workers in a firm to motivate their work effort can be lower if the workers feel like they are insiders. That they identify with the goals of the firm and feel part of a team of insiders boosts their productivity and raises the firm's profitability. In that sense, firms that devote efforts to help its employees "feel part of a team" are undertaking investments with future payoffs. No doubt, the reasons that indigenous citizens feel that they are part of a team while immigrants taking on citizenship are not part of the same team are complex. For example, having citizen parents share in the country's struggles to transit from Third World to First, an experience not shared by new immigrants or their parents, foster a sense of identity among indigenous citizens. As a result, when an immigrant receives the same privileges as an indigenous citizen, the latter may experience a loss of utility because of a loss of the sense of identity. ("There is nothing special about being a Singaporean.") As the share of immigrants in the citizen population size increases, the loss of social capital can be counted as a cost to society that could rise at an increasing rate.

The optimal population mix for a given increase in citizen population size is achieved when the marginal cost of getting another headcount via measures to boost total fertility is equal to the marginal cost of getting another headcount via net immigration. An investment to forge a national identity that recognises the economic contributions of both indigenous citizens and immigrants can lower the marginal cost curve from achieving population increase via net immigration and thus reduce the total cost of achieving a given population size increase. An environment that enhances work-life balance can simultaneously lower the marginal cost curve of achieving population increase via measures to boost total fertility.

In what follows, we present a more formal discussion of the welfare gains or losses from population increase. We then develop more fully the arguments leading to the optimal mix of the two ways to achieve a population size increase. The main findings of the paper can be summarised in the form of 10 takeaways:

1. Neoclassical growth theory, with physical capital accumulation being a major driving force behind the rise in standard of living, predicts that, with diminishing marginal product of capital, a decline in population growth rate,

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10. A key paper of theirs is Akerlof and Kranton (2000).

holding labour force participation rate constant, leads to a rise in the standard of living due to a "capital dilution" effect.

2. The rate of population growth that can be sustained without placing severe limits on the growth in the standard of living due to limited supply of usable land depends positively on the rate of technological progress and negatively on the share of land as a factor of production. However, even if technological progress could mitigate the diminishing marginal returns to labour due to limited supply of land, too high a population density is likely to lead to strong "congestion" effects that make it desirable to have a smaller population size.

3. As the technology gap has narrowed, and huge investments in education have raised the average level of human capital of workers, the standard of living in a future where citizen population size is down, say, to the level it was in 1970, is likely to remain higher than it was in 1970.

4. As Singapore's technology gap has narrowed due to past decades of catch-up, facilitated by free international trade and inflows of multinational corporations, its future (more modest) growth in standard of living must be fueled by indigenous innovations. If creativity is important for innovation, and the probability of being a creative and talented person is independently distributed, then a greater number of creative and talented people are present in a larger population. This "Mozart effect" channel and a "human capital externalities" channel make it desirable to have a larger population size.

5. At any point in time, there co-exist different generations of people within a given population. There are a certain number of young people who work and who have the capacity to pay taxes that fund items such as health subsidies that are predominantly received by a certain number of old and retired people. A decline in the population size reduces the "tax base" supporting each old and retired person, making it desirable to increase population growth.

6. The reputation that Singapore has built as a good investment destination allows it to grow its labour supply (by boosting population size) without a corresponding decline in labour productivity because capital grows to support the additional headcounts. The higher GDP growth, in turn, lowers the unemployment rate based on "Okun's Law."

7. As individuals become richer, earning a higher market wage per unit of time spent working, the opportunity cost of the time devoted to raising a child correspondingly increases, which leads couples to choose to have fewer children and to make bigger human capital investments in each child. (This can be called a "level" effect.) Policy measures such as parental leave, childcare subsidy and the Baby Bonus effectively lower the opportunity cost of having an additional child and produce substitution and income effects.

8. In an era of high growth rates, the potential to gain from parental investment in a child's education might be higher as higher skills are more valued in a rapidly growing economy. If this is true, then the implied growth slowdown as the Singapore economy matures could mean that total fertility

rate would increase on this account. Whether this "growth" effect will offset the "level" effect, thus reversing the trend toward lower total fertility rate, is an empirical question.

9. While the cost of policy measures to boost the total fertility rate can be explicitly accounted for in the national budget, the cost to society of achieving a given increase in population size via net immigration appears to be more indirect and works through its effect on social capital. An optimal mix of the measures to boost total fertility, on the one hand, and net immigration, on the other hand, to achieve a given increase in citizen population size equates the marginal cost of both approaches.

10. An investment to forge a national identity that recognises the economic contributions of both indigenous citizens and immigrants can lower the marginal cost curve from achieving population increase via net immigration and thus reduce the total cost of achieving a given population size increase. An environment that enhances work-life balance can simultaneously lower the marginal cost curve of achieving population increase via measures to boost total fertility.

Arriving at the Optimum Population Size

Capital dilution effect

To understand the "capital dilution" effect resulting from a higher population growth rate, it is convenient to begin with the Solow neoclassical growth model abstracting from technological progress and role for human capital accumulation. We take the Cobb-Douglas production function, $Y_t = K_t^{\alpha} L_t^{1-\alpha}$; $0 < \alpha < 1$, where Y_t is total real GDP (or output), K_t is physical capital stock, L_t is total number of workers (equal to the size of labour force). For simplicity, we treat the labour force participation rate as exogenously given and normalise it to one. Dividing the output by number of workers, we get $y_t \equiv Y_t/L_t = k_t^{\alpha}$, where y_t is output per worker and $k_t \equiv K_t/L_t$ is the capital per worker or measure of capital intensity.

We assume that the aggregate consumption function is simply $C_t = (1-s)Y_t$, where $0 < s < 1$ is the savings rate. It also follows that the savings function is simply

$$S_t = sY_t \tag{1}$$

Aggregate gross investment, I_t is given by

$$I_t = K_{t+1} - (1-\delta)K_t, \tag{2}$$

where δ is the rate of physical capital depreciation.

For the goods market to be in equilibrium, we must have that aggregate demand equals output. We let aggregate demand be simply given by $C_t + I_t$.

Hence, $Y_t = C_t + I_t$. The goods market equilibrium condition can be re-expressed as $S_t = I_t$, from which, after substitutions, we obtain

$$K_{t+1} - K_t = sK_t^\alpha L_t^{1-\alpha} - \delta K_t.$$ (3)

Dividing through equation (3) by L_t, and re-arranging, we get

$$nk_{t+1} = sk_t^\alpha + (1-\delta)k_t,$$ (4)

where $L_{t+1} = nL_t$ so n is the gross population growth rate. Further dividing both sides of equation (4) by n and subtracting k_t from both sides, we get, after re-arranging, the fundamental equation of growth in the Solow neoclassical growth model:

$$k_{t+1} - k_t = \frac{sk_t^\alpha - (n-1+\delta)k_t}{n}.$$ (5)

In steady state, $k_{t+1} - k_t = 0$ so

$$k_{ss} = \left(\frac{s}{n-1+\delta}\right)^{\frac{1}{1-\alpha}}.$$ (6)

where we use the subscripts ss to represent a steady-state value. Noting that $y_{ss} = k_{ss}^\alpha$, we get

$$y_{ss} = \left(\frac{s}{n-1+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$ (7)

The central results are in equations (6) and (7). From equation (6), we learn that an increase in the growth rate of population, n, results in a decrease in k_{ss}, the "capital dilution" effect. As a result of a lower k_{ss}, we find from equation (7) that the output per worker or standard of living declines with an increase in population growth rate. Thus, we have proved our first takeaway:

Takeaway Number 1: *Neoclassical growth theory, with physical capital accumulation being a major driving force behind the rise in standard of living, predicts that, with diminishing marginal product of capital, a decline in population growth rate, holding labour force participation rate constant, leads to a rise in the standard of living due to a "capital dilution" effect.*

Constraint of fixed supply of land

To see how much limited land acts as a constraint on the optimum population growth rate, let us introduce land as an explicit factor of production. We let the production function be given by

$$Y_t = K_t^\alpha (A_t L_t)^\beta T^{1-\alpha-\beta},$$ (8)

where T is the fixed supply of land and A_t is an index of technology applied in the economy. We assume that $\alpha > 0,\ \beta > 0,\ \alpha + \beta < 1.$ If, for convenience, we work with continuous time, and use the dot notation to represent the time rate of change (so $\dot{x} \equiv dx_t/dt$), the factors of production evolve over time according to the following laws of motion:

$$\dot{K}_t = sY_t - \delta K_t, \tag{9}$$

$$\dot{L}_t = g_L L_t, \tag{10}$$

$$\dot{A}_t = g_A A_t, \tag{11}$$

where g_L denotes the instantaneous rate of growth of population and g_A is the instantaneous rate of growth of technology. Solving the system of equations given by (8) to (11), it can be readily shown that the instantaneous rate of growth of output per worker, $g_{Y/L}$ is given by the following:

$$g_{Y/L} = \frac{\beta g_A - (1 - \alpha - \beta) g_L}{1 - \alpha}. \tag{12}$$

While technological progress, denoted by g_A, can sustain growth in output per worker despite the fixity of land, a measure of population density, represented by L_t/T, can be taken to cause inconvenience to individuals resulting from overcrowding and congestion thus placing a limit on the optimum size of the population. Thus, the second takeaway is:

Takeaway Number 2: *The rate of population growth that can be sustained without placing severe limits on the growth in the standard of living due to limited supply of usable land depends positively on the rate of technological progress and negatively on the share of land as a factor of production. However, even if technological progress could mitigate the diminishing marginal returns to labour due to limited supply of land, too high a population density is likely to lead to strong "congestion" effects that make it desirable to have a smaller population size.*

Education and technological progress

We can now broaden the neoclassical growth model to incorporate human capital accumulation and technological progress. (To simplify, we put aside the constraint of land and leave it out as a factor of production.) Again, we set the model in continuous time. Output at time t can be written as

$$Y_t = K_t^\alpha \left[A_t h_t L_t \right]^{1-\alpha}, \tag{13}$$

where h_t is the average level of human capital. Following Jones (2002), we write $h_t = e^{\phi E}$, $\phi > 0$, where E is the number of years spent schooling. [11] Equation (13) can be solved along with equations (9) to (11) giving the fundamental equation of growth in the augmented Solow model (in continuous time) as

$$\dot{k}_t = sk_t^\alpha - (g_L + g_A + \delta)k_t, \tag{14}$$

where $k_t \equiv K_t / \left[A_t e^{\phi E} L_t \right]$. In steady state, $\dot{k}_t = 0$ so we obtain

$$k_{ss} = \left(\frac{s}{g_L + g_A + \delta} \right)^{\frac{1}{1-\alpha}}. \tag{15}$$

We can write the steady-state output per worker at time t, $(Y/L)_{ss}(t)$, which itself is growing at the rate of growth of technology, g_A, as

$$\left(\frac{Y}{L} \right)_{ss}(t) = A(t) e^{\phi E} \left(\frac{s}{g_L + g_A + \delta} \right)^{\frac{\alpha}{1-\alpha}}. \tag{16}$$

To conduct our thought experiment of returning our population size in the year $2012 + T$ back to the population size prevailing in 1970, equation (16) tells us that we would still be richer in the year $2012 + T$ because of technological progress and increase in the average number of years of schooling. Thus, the third takeaway is:

Takeaway Number 3: *As the technology gap has narrowed, and huge investments in education have raised the average level of human capital of workers, the standard of living in a future where citizen population size is down, say, to the level it was in 1970, is likely to remain higher than it was in 1970.*

Mozart effect and human capital externalities

How did Singapore transit from Third World to First? Looking at the model described by equations (13) to (16) suggests at least three factors: a high savings rate (s), an increase in the average number of years of schooling (E), and technological catch-up (g_A). We can make some progress in modelling the process of technological catch-up. Suppose that there is an index of technology of world technological leaders (frontier technology) described by $A^*(t)$ at time t that is the result of past $R\&D$ investments undertaken by the technological leaders. We can describe Singapore's time rate of change of technology, $\dot{A}(t)$, as proportional to the size of the technology gap, $A^*(t) - A(t)$. Hence, we write

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11. This specification is based upon the microeconomic evidence, which suggests that each additional year of education increases an individual's wage by approximately the same percentage amount (Romer 2012, 153).

$$\dot{A}(t) = \mu L(t)\left[A^*(t) - A(t)\right] \text{ if } A^*(t) > A(t),\tag{17}$$

$$\dot{A}(t) = 0; \text{ otherwise,}\tag{18}$$

where μ is the share of the labour force in Singapore devoted to learning the technology of the world leaders. So long as a positive technological gap exists, the implication of the model is that, in the steady state, the ratio $A^*(t)/A(t) > 1$ is constant with Singapore's rate of growth of technology being equal to the rate of growth of technology among technological leaders.

It might be argued, however, that, in the next phase of Singapore's growth, it has to transit from being a technology follower to becoming a technological leader, creating its own innovative products. Then, the "Mozart effect" and "human capital externalities" channel would suggest that it is desirable to have a bigger population size. Thus, the fourth takeaway is:

Takeaway Number 4: *As Singapore's technology gap has narrowed due to past decades of catch-up, facilitated by free international trade and inflows of multinational corporations, its future (more modest) growth in standard of living must be fueled by indigenous innovations. If creativity is important for innovation, and the probability of being a creative and talented person is independently distributed, then a greater number of creative and talented people are present in a larger population. This "Mozart effect" channel and a "human capital externalities" channel make it desirable to have a larger population size.*

An overlapping-generations model

Perhaps the most compelling reason for being concerned about a declining population accompanied by increased longevity is the reduced number of young workers who form the tax base to fund expenditures such as health subsidies that go disproportionately to the old and retired. One of the important insights from the overlapping-generations model introduced by Paul Samuelson is that the welfare of individuals in a society over a lifetime can be raised through an implicit social contract where the young pay taxes to finance benefits received by the old. A young person's lifetime utility would be increased if the gross rate of population growth exceeds the gross world interest rate even though he or she pays taxes while working because the next generation of young workers will finance his or her benefits when old.

Let an individual live for two periods of life. When young, he or she works and receives an income of y. When old, he or she is retired. Lifetime utility is given by $u(c^y) + (1 + \rho)^{-1} u(c^o)$, where ρ is the subjective rate of time preference, c^y is consumption when young, and c^o is consumption when old. First-period budget constraint is given by $c^y + s = y - \tau^{tax}$, where s is private saving and τ^{tax} is the amount of tax paid when young. The second-period

budget constraint is given by $c^o = rs + b$, where r is the exogenously given world gross rate of interest and b is the amount of benefits received when old. Consolidating the first- and second-period budget constraints gives a consolidated lifetime budget constraint:

$$c^y + \frac{c^o}{r} = y - \tau^{tax} + \frac{b}{r}. \tag{19}$$

Under a balanced-budget policy, the government budget constraint is given by $L_t \tau^{tax} = L_{t-1} b$. Since $L_t = nL_{t-1}$, we can re-express the government budget constraint as $\tau^{tax} = b/n$. Substituting into equation (19) gives

$$c^y + \frac{c^o}{r} = y + \left[\frac{n-r}{nr}\right]b. \tag{20}$$

Equation (20) tells us that an individual's lifetime utility is increased as his or her lifetime income, taking into account taxes and benefits received and evaluated at the world interest rate, is higher the higher the gross rate of population growth. Thus, the fifth takeaway is:

Takeaway Number 5: *At any point in time, there co-exist different generations of people within a given population. There are a certain number of young people who work and who have the capacity to pay taxes that fund items such as health subsidies that are predominantly received by a certain number of old and retired people. A decline in the population size reduces the "tax base" supporting each old and retired person, making it desirable to increase population growth.*

Okun's Law

We have so far abstracted from unemployment. However, at any point in time, due to wage bargaining or efficiency wage considerations, the volume of unemployment is positive. There exists a strong positive historical relationship between Singapore's total real GDP growth rate (above a minimum level) and a change in the unemployment rate dubbed "Okun's Law." As Singapore has developed a business environment that has been friendly to both foreign and domestic investments by private firms, an enlargement of the pool of available workers need not lead to higher unemployment. Instead, as capital investments occur in tandem with an enlarged labor force size, total real GDP growth occurs with a decline in the unemployment rate based on Okun's Law. Thus, the sixth takeaway is:

Takeaway Number 6: *The reputation that Singapore has built as a good investment destination allows it to grow its labour supply (by boosting population size) without a corresponding decline in labour productivity because capital grows to support the additional headcounts. The higher GDP growth, in turn, lowers the unemployment rate based on "Okun's Law".*

Summing up of this section

We have considered two channels that would suggest that more rapid
population growth lowers the utility of a typical citizen, namely, the "capital
dilution" channel and the "congestion" channel. On the other hand, we
identified four other channels that would suggest that a more rapid growth of
the citizen population size is desirable, namely, the "tax base" channel, the
"Mozart effect" channel, the "human capital externalities" channel, and the
"Okun's Law" channel. In principle, if we express the typical citizen's utility as
a function of the gross rate of population growth, n, welfare is a reduced-form
function of n through

$$W(n) = F(f_1(n), f_2(n), f_3(n), f_4(n), f_5(n), f_6(n)); \tag{21}$$

$$f_1'(n) < 0; \quad f_2'(n) < 0: \quad f_3'(n) > 0; \quad f_4'(n) > 0; \quad f_5'(n) > 0; \quad f_6'(n) > 0,$$

where a prime represents a derivative taken with respect to n. The derivative
$f_1'(n) < 0$, for example, represents the "capital dilution" channel. If $W(n)$ is
strictly concave, with $W''(n) < 0$, the socially optimal gross rate of growth of
population is given by n^{opt} where

$$W'(n^{opt}) = 0. \tag{22}$$

If we are willing to make the judgment that the path of optimum population
size determined in equation (22) lies above the path implied by a total fertility
rate of 1.2 without net immigration, the next question is: how should we
achieve the increase in citizen population — through measures to boost the
total fertility rate or through net immigration? We turn next to this issue.

Optimal Mix to Achieve Population Size Increase

Measures to boost total fertility

We suppose that a two-person household (excluding children) solves the
following utility maximization problem:[12] Maximize

$$(1 - \gamma) \ln c_t + \gamma \left[\ln n_t + \beta \ln h_{t+1} \right]$$

subject to

$$h_{t+1} = h(e_{t+1}, g_{t+1}), \tag{23}$$

$$n_t(\tau + \tau^e e_{t+1}) w_t^w + c_t = w_t^w + w_t^m + B n_t, \tag{24}$$

where c_t is household consumption, n_t is the number of children, h_{t+1} is
human capital, e_{t+1} is the education required to produce human capital, g_{t+1} is
the economy's gross growth rate, τ is the fixed time cost of raising each child
that is independent of education, τ^e is the time cost associated with

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12. The presentation follows the particular formulation adopted by Galor (2011) who expressed the cost
of raising children in terms of time cost and who introduced a function relating human capital
accumulation to education and the economy's growth rate.

educating a child, w_t^w is the woman's market wage, w_t^m is the man's market wage, and B is the Baby Bonus given to each new child. Note that β gives the weight parents place on the human capital of each child.

Substituting out for c_t using equation (24), we obtain two first-order conditions from choosing optimal n_t and e_{t+1} (the "quantity" and "quality" choice), respectively:

$$\frac{\left[(\tau + \tau' e_{t+1})w_t^w - B\right]n_t}{w_t^w + w_t^m - \left[(\tau + \tau' e_{t+1})w_t^w - B\right]n_t} = \frac{\gamma}{1-\gamma}, \tag{25}$$

$$\frac{w_t^w \tau^e}{(\tau + \tau^e e_{t+1})w_t^w - B} = \frac{\beta h_e(e_{t+1},g_{t+1})}{h(e_{t+1},g_{t+1})}, \tag{26}$$

where $h_e(e_{t+1},g_{t+1})$ is the partial derivative with respect to the first argument. We adopt the following properties for the $h(e,g)$ function: $h_e > 0$, $h_{ee} < 0$, $h_{eg} > 0$, and $h_g < 0$. We assume that $B < (\tau + \tau^e e_{t+1})w_t^w$, that is, the Baby Bonus is less than the total opportunity cost of having a child.

From equation (25), we can prove the following results:

Result 1: $\partial n/\partial w^w < 0;\ \partial n/\partial w^m > 0;\ \partial n/\partial g < 0;\ \partial n/\partial \beta < 0;\ \partial n/\partial B > 0;$ $\partial n/\partial \tau < 0:\ \partial n/\partial \tau^e < 0.$

Result 1 tells us several predictions from the Becker-Galor fertility model on *ceteris paribus* assumptions. On the assumption that it is the woman who directly devotes time to child-raising, we find that an increase in the woman's wage (w^w) leads to reduced fertility as the opportunity cost of time spent raising children increases and the substitution effect dominates. An increase in the man's wage (w^m), however, has a pure income effect so that its rise increases the number of children the couple chooses to have. Higher growth (g) and a higher weight placed on a child's human capital (β) lead to fewer children as parents choose "quality" over "quantity" and devote more resources to education.[13] A Baby Bonus given on a per child basis lowers the opportunity cost of having children and leads to increased fertility. Note that the effect of parental leave is analogous to the effect of a reduction in the fixed time cost of raising a child regardless of the level of human capital; it raises fertility.

An educational subsidy has effects analogous to the reduction of τ^e; it also raises fertility.

From equation (26), we can prove the following results:

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13. This channel will be confirmed in Result 2.

Result 2: $\partial e / \partial g > 0$; $\partial e / \partial \beta > 0$; $\partial e / \partial B < 0$; $\partial e / \partial \tau > 0$.

Result 2 tells us that a higher growth rate (g) raises the return to parental investment in education and thus encourages parents to invest more in education, choosing "quality" over "quantity". Similarly, a stronger preference for human capital (β) leads to a choice of "quality" over "quantity". On the other hand, a Baby Bonus and a reduction of the fixed time cost of raising a child, such as through parental leave, lead to a choice of having more children with a substitution away from "quality." Thus, the seventh and eighth takeaways are:

Takeaway Number 7: *As individuals become richer, earning a higher market wage per unit of time spent working, the opportunity cost of the time devoted to raising a child correspondingly increases, which leads couples to choose to have fewer children and to make bigger human capital investments in each child. (This can be called a "level" effect.) Policy measures such as parental leave, childcare subsidy and the Baby Bonus effectively lower the opportunity cost of having an additional child and produce substitution and income effects.*

Takeaway Number 8: *In an era of high growth rates, the potential to gain from parental investment in a child's education might be higher as higher skills are more valued in a rapidly growing economy. If this is true, then the implied growth slowdown as the Singapore economy matures could mean that total fertility rate would increase on this account. Whether this "growth" effect will offset the "level" effect, thus reversing the trend toward lower total fertility rate, is an empirical question.*

Net immigration

Unlike the measures to boost total fertility whose total cost can be measured, the cost of increasing one more person to the citizen population pool via net immigration is harder to measure. Holding other things constant, a smaller Baby Bonus is required to induce a couple to have another child if their preference for human capital ("quality") as measured by β is smaller than that of another couple. If we suppose that we can rank couples by the values of their β in an increasing order, we can derive an upward-sloping marginal cost curve against the number of new citizens coming from boosting total fertility. Apart from any explicit fiscal costs incurred in attracting new immigrants to take up Singapore citizenship, it appears that the cost to society of expanding the population via net immigration takes the form of a loss of social capital. Akerlof and Kranton (2000) introduce the concept of identity in economic decision-making that is useful to make sense of how net immigration can cause a loss of social capital. Writing individual j's utility as $U_j(a_j, a_{-j}, I_j)$, where a_j represents actions undertaken by individual j, a_{-j} the actions undertaken by members other than j, and I_j is individual j's identity, we express the idea that individual j's identity comes from the social category with which he identifies.

An indigenous citizen whose own parents are Singapore citizens might identify with other indigenous citizens and feel that they "belong to the same team". Such feelings may not extend to new immigrants who are seen as "outsiders". An increase in the share of new immigrants in a given pool of expanded population size might lead to a loss of identity by indigenous citizens which make them less willing to co-operate in the production activity or less willing to share in bearing the costs of national programmes. The loss of social capital might increase at an increasing rate as the share of new immigrants increases giving rise to an upward-sloping marginal cost curve against the number of new immigrants.

Suppose that it is calculated that it is desirable to increase citizen population size by ΔL. How many of ΔL should come from bearing the costs of boosting total fertility and how many should come from net immigration. We want to find the optimal number of indigenous citizens that come from measures to boost total fertility, denoted $L_{indigenous}^{opt}$, and the optimal number of new immigrants, denoted $L_{immigrant}^{opt}$, such that

$$L_{indigenous}^{opt} + L_{immigrant}^{opt} = \Delta L. \tag{27}$$

Let the marginal cost from adding another indigenous citizen be given by $MC(L_{indigenous})$; $MC'(L_{indigenous}) > 0$ and the marginal cost from adding another immigrant be given by $MC(L_{immigrant})$; $MC'(L_{immigrant}) > 0$. Then, the optimal number of new immigrants is given by

$$MC(L_{immigrant}^{opt}) = MC(\Delta L - L_{immigrant}^{opt}). \tag{28}$$

The ninth takeaway can be expressed as:

Takeaway Number 9: *While the cost of policy measures to boost the total fertility rate can be explicitly accounted for in the national budget, the cost to society of achieving a given increase in population size via net immigration appears to be more indirect and works through its effect on social capital. An optimal mix of the measures to boost total fertility, on the one hand, and net immigration, on the other hand, to achieve a given increase in citizen population size equates the marginal cost of both approaches.*

The marginal cost curves can themselves be shifted. National efforts to integrate immigrants can shift down the $MC(L_{immigrant})$ curve. Similarly, a better work-life balance can shift down the $MC(L_{indigenous})$ curve. Such simultaneous shifts of the marginal cost curves mean that the total cost to society of achieving the desired increase in citizen population size, ΔL, can be reduced. The cost savings can then go toward fulfilling other national objectives. We have the tenth takeaway:

Takeaway Number 10: *An investment to forge a national identity that recognises the economic contributions of both indigenous citizens and*

immigrants can lower the marginal cost curve from achieving population increase via net immigration and thus reduce the total cost of achieving a given population size increase. An environment that enhances work-life balance can simultaneously lower the marginal cost curve of achieving population increase via measures to boost total fertility.

Concluding remarks

Singapore went against the odds to make a transition from Third World to First within a few decades. As a mature economy, the average Singapore citizen is richer today than a few decades ago. A declining fertility rate, itself a consequence of success in generating economic prosperity, and increased longevity imply that the citizen old-age support ratio will drastically fall if there is no rise in fertility rate and net immigration. An optimal mix to achieve a given increase in citizen population size most likely requires both measures to boost total fertility as well as net immigration. Non-economic factors such as family ties aside, since countries all over the world typically grant citizenship to skilled individuals, the relative share of skilled workers in the economy is likely to increase. This can act to further widen income inequality. Since both the new immigrants as well as the receiving country stand to gain from the immigration, it is important that there be a willingness to use some of the social surplus generated to fund wage subsidies through a national programme like the Workfare Income Supplement (WIS) scheme and Workfare Training Support (WTS) scheme to boost the pay and employment of low-wage workers.

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