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Cross-licensing in Vertically Related Markets*

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Abstract

We study the market impacts of a dominant upstream firm signing a compulsory zero-royalty cross-licensing agreement with downstream competing producers. Such a business strategy, typically referred to as reverse licensing, can allow the dominant firm to extend its market power from the upstream market to the downstream innovation and production markets. We show that reverse licensing in vertically related markets in general achieves static efficiency but can result in dynamic inefficiency. Under fairly plausible conditions, reverse licensing will raise the profit of the dominant upstream firm, but lower the profits of downstream producers, consumer surplus and welfare. While this paper extends the classical complementary inputs problem to a vertical setting, it also provides a theoretical framework to evaluate the influential antitrust case of Qualcomm. This framework can also provide a rationale for the remedy adopted by China's competition authority on Qualcomm and its reverse licensing practice.

***JEL* Classification:** D45, L40, O3

Keywords: Reverse Licensing; Vertical Relation; Welfare; Patent Pool; Research Incentive; Complement problem; Pass-through rights

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1 Introduction

In today's rapidly evolving research landscape, the well-defined boundaries of technologies between upstream and downstream firms in vertically related markets have undergone a remarkable transformation. The symbiotic relationship between these entities has extended beyond conventional supply-chain dynamics, driven by the integration of advanced technologies.

While traditional analyses of technology transfers among firms predominantly focus on horizontal markets (e.g., Arrow, 1972; Kamien, 1992; Vives, 2008), this paper aims to delve into the intricate technological relationship between upstream manufacturers and downstream producers. In this context, the upstream firm assumes a dual role: supplying intermediary inputs to downstream counterparts and actively participating in and facilitating technology transfer along the supply chain.

Additionally, since we acknowledge the heterogeneity among downstream firms, with some possessing process innovations resulting from prior research and development (R&D) efforts, our research not only explores technology transfer strategies between upstream and downstream firms but also investigates the product competition strategies employed along the supply chain and the interactions between them.

Understanding the complex interplay of technology and innovation within the context of upstream manufacturers and downstream producers holds significant relevance for academia and industry alike. From an academic perspective, this research contributes to the expanding literature on technology transfer, inter-organizational relationships, and innovation-driven business models (Mansfield, 1986; Pack and Saggi, 2001; Hu et al., 2005; Blalock and Gertler, 2008; Hegde et al., 2009). From a practical standpoint, the insights gained from this study can inform decision-making processes and strategic initiatives aimed at optimizing technology-enabled collaborations, enhancing operational efficiency, and fostering competitive advantage.

Moreover, the intensifying competition and collaboration in research and development have given rise to a prevalent practice of patent pool (Lerner and Tirole, 2004; Lerner et al., 2007), cross-licensing agreements (Fershtman and Kamien, 1992; Choi and Gerlach, 2019) and the establishment of Standard Setting Organizations (Chiao et al., 2007; Rysman and Simcoe, 2008; Sidak, 2013). These initiatives aim to address interoperability and compatibility challenges but concurrently introduce new complexities such as royalty-stacking and hold-up problems (Shapiro, 2000; Lemley and Shapiro, 2006). While scholars have proposed various solutions to tackle these challenges specifically within horizontal settings (e.g., Lerner and Tirole, 2015; Schmalensee, 2009; Hougaard et al., 2023), our research shifts its focus to a distinct concern within the vertical structure.

An illustrative real-world instance that exemplifies the implications of this issue is the influential antitrust case involving Qualcomm Incorporated (Qualcomm) in China. Qualcomm, a global leader in smartphone chip manufacturing,[1] held a near-monopoly position in China's market in 2013, dominating key aspects of wireless communications standard essential patents (SEPs).[2] Qualcomm also had 100% market share in the licensing of relevant wireless communications SEPs and above 50% market share in CDMA, WCDMA, and LTE baseband chip markets. China's National Development and Reform Commission (NDRC), the authority responsible for addressing competition-related violations of China's Anti-Monopoly Law, commenced an investigation into possible exploitation of Qualcomm's dominant market position, starting in November 2013.[3] After a period of 17 months, the NDRC, on March 2, 2015, reached the conclusion that Qualcomm's conduct had indeed contravened antitrust regulations. As a result, Qualcomm was mandated to cease its anti-competitive practices and was issued a substantial fine of RMB 6.088 billion (approximately US$975 million).[4] Qualcomm announced its decision not to contest the NDRC's ruling and agreed to modify certain aspects of its patent licensing and baseband chip sales practices in China.

Among the several anti-competitive conducts adopted by Qualcomm, one notable practice is called *reverse licensing*. This refers to the requirement imposed on Chinese licensees to cross-license their relevant SEPs and non-SEPs to Qualcomm and its customers without compensation or offsetting royalties.[5] This is our main contribution, as this practice has not been formally studied in the literature.

Our model shows that "reverse licensing" would foster product market competition but hinder R&D efforts along the supply chain. It is interesting to see if this actually aligns with the causal observation from the Qualcomm case.

In China's smartphone market, downstream producers are rather heterogeneous. The principal downstream producers in China's smartphone market are Huawei Technologies Co. Ltd (referred to

[1]According to the Strategy Analytics report, Qualcomm is the leader with a share of 42% in the global smartphone processor market.

[2]A standard-essential patent is a patent that claims an invention that must be used to comply with a technical standard (Shapiro, 2000).

[3]China is not the only country investigating Qualcomm's anticompetitive conduct. Prior to reaching an agreement in 2008, Nokia complained to Qualcomm about charging expired patents and high royalty rates. In 2009, South Korea fined Qualcomm 260 billion Won (USD 207 million) for abusing its market dominance. More recently, in 2015, the EU started an investigation into predatory pricing, where Qualcomm allegedly drove its competitors out of the market

[4]According to Article 47 of the AML, the relevant antimonopoly enforcement authority has the power to order the operator to cease objectionable activities, confiscate illegal gains, and impose fines ranging from 1% to 10% of the turnover from the previous year.

[5]The other anti-competitive behaviours include unjustified bundling of SEPs and non-SEPs, imposition of unreasonable sales terms on baseband chip customers, charging royalties for expired patents, etc.

as “Huawei”), Zhongxing Telecommunication Equipment Corporation (referred to as “ZTE”), Xiaomi Inc. (referred to as “Xiaomi”), and OPPO Electronics Corp. (referred to as “OPPO”). Notably, Huawei and ZTE held substantial patent portfolios with approximately 30,000 and 52,000 patents, respectively, while Xiaomi and OPPO held fewer patents, numbering 10 and 103, respectively.[6].

However, their market shares are not too different from each other. According to a report from the analytical firm TrendForce, Huawei and ZTE held market shares of 8.4% and 3.1% respectively in 2015. Despite their limited patent counts, Xiaomi and OPPO commanded market shares of 5.6% and 3.8% respectively.[7]

Qualcomm’s practice of “reverse licensing,” wherein Huawei and ZTE were required to relinquish patents without charge upon chip purchase, and Xiaomi and OPPO obtained chips along with complimentary patents, raised concerns about the potential dampening of innovation incentives across all participants. Particularly, Huawei and ZTE seemed to derive no benefits from their research efforts, while Xiaomi and OPPO could potentially capitalize on the efforts of others.[8] However, while the practice of reverse licensing might have hindered R&D efforts, it could also have fostered competition within the Chinese smartphone market. Our model aligns with this observation, illustrating the positive effects of reverse licensing on prices, quantities, and consumer surplus.

To facilitate our research purpose, we develop a theoretical model in a supply chain setting where an upstream monopolist (Qualcomm) sells an intermediary good (smartphone chips) to competing downstream firms that produce final goods (smartphones) sold to consumers. Downstream firms are heterogeneous in their production technologies: leading firms (such as Huawei) have advanced technologies (or lower production costs) and the rest of the firms have standard technologies of production.

To evaluate the potential impacts of reverse licensing, we consider a four-stage game-theoretical model. In the first stage, the upstream firm sets a retail price for the intermediary input. In the second stage, downstream firms independently conduct R&D. In the third stage, technology transfers among downstream firms may take place depending on whether reverse licensing is imposed by the upstream firm. Downstream firms engage in Cournot competition in the last stage.

We find that reverse licensing that forces downstream firms to cross-license their technologies for free can generate several important effects. First, reverse licensing tends to relax competition

[6]Source: Di Yi Cai Jing Ri Bao, 03 December 2014. Retrieved from “http://tech.sina.com.cn/t/2014-12-03/02259846574.shtml” on 04 December 2014.

[7]Retrieved from ”http://press.trendforce.com/press/20160114-2265.html” on 08 June 2016.

[8]More recently, the Chinese government introduced a new development plan that underscores the significance of fostering innovation. One of the main aims of investigating Qualcomm’s practices is to promote innovation while safeguarding the interests of local smartphone manufacturers.

in the downstream market, which enables the dominant upstream firm to raise the input price and strengthen its market power (pricing effect). Second, reverse licensing fully eliminates weak firms' innovations (since they can free ride on others' innovation efforts) and also lowers leading firms' innovation levels, and hence weakens downstream firms' incentives to innovate (innovative effect). Third, reverse licensing can improve productive efficiency by ensuring the full diffusion of the most efficient technology in the downstream market (competitive effect). While the first two effects tend to raise final prices or lower the total output, the third effect is generally efficient in the sense of reducing final prices. These countervailing effects indicate that reverse licensing can cause ambiguous impacts on firms' profits, consumer surplus and welfare.

First, from the perspective of the upstream firm, while the competitive effect is in favour of raising its profit, the innovative effect goes against it. Hence, reverse licensing will make the upstream firm better off when the competitive effect dominates the innovative effect. This occurs when the cost asymmetry among downstream firms is sufficiently high (so the competitive effect is strong) and the cost of R&D is generally high (so the innovative effect is weak). As for downstream firms, while reverse licensing always benefits weak firms, it can only benefit leading firms when the cost asymmetry among downstream firms is sufficiently low (so that these firms do not hurt much by the competitive effect). Second, reverse licensing will also lead to higher consumer surplus, when the cost asymmetry among downstream firms is sufficiently high and the cost of R&D is sufficiently high so that the positive innovative effect dominates the other two negative effects (i.e., pricing effect and innovative effect). Third, since social welfare is the sum of producer surplus and consumer surplus, reverse licensing may also raise or reduce social welfare under certain conditions. In particular, we show that under some plausible conditions, reverse licensing will raise the profit of the upstream firm so that it will be adopted as an optimal strategy. However, reverse licensing may be anti-competitive in the sense that it results in lower consumer surplus and total welfare.

Next, we study the model under the regulation where the upstream manufacturer cannot force downstream producers to surrender the technologies. To incentivise downstream producers to cross-license their technologies, the upstream producer needs to compensate them adequately. We model this by allowing the upstream firm to offer a royalty compensation scheme to downstream firms that license their technologies to others. We find that as compared to the case of royalty-free reverse licensing, such a remedy tends to improve downstream firms' R&D incentives and raise consumer surplus. Surprisingly, when the cost asymmetry among downstream firms is sufficiently large, the upstream firm can also obtain a higher profit even though it has to compensate downstream firms through royalty fees. These findings can provide theoretical support for the regulation by the Chinese government in the Qualcomm antitrust case, as such remedies will not only protect the innovation incentives of the domestic industry and improve consumer welfare, but may also be beneficiary to the upstream firm.

To show the robustness of our key findings, we further extend our baseline setting in several important directions. For instance, we consider more general R&D behaviours of downstream firms by analyzing situations with general compatible technologies, general degree of technology spillovers, and different R&D technologies of downstream firms. We also consider more general demand structures (e.g., model of differentiated products) or more general market structures (e.g., oligopoly downstream market with multiple competing downstream firms). In addition, we also explore models in which downstream firms may engage in independent licensing in the absence of reverse licensing, the upstream firm may also undertake R&D, etc. While our findings tend to suggest that the key mechanism in the baseline model continues to be valid, some other factors (e.g., product differentiation, downstream market structure, compatibility of technologies) can also play an important role in quantifying the impacts of reverse licensing on market participants and welfare.

There is a small literature on cross-licensing agreements. Fershtman and Kamien (1992) analyze how cross-licensing affects the pace of the innovation race. Jeon and Lefouili (2018) analyze the competitive effects of cross-licensing and show that bilateral cross-licensing agreements can result in a monopoly outcome. Choi and Gerlach (2019) study the optimal cross-licensing agreements between incumbent firms that are faced with potential entrants. Unlike these papers, which focus on cross-licensing agreements in horizontal markets, we consider cross-licensing imposed by a dominant upstream firm on downstream producers in vertically related markets.

The closest work to our paper is Wang et al. (2023), which also considers cross-licensing in a supply chain setting. They analyze whether or not an upstream supplier should commit to cross-licensing and further study the impacts of cross-licensing on firms' profits, consumer surplus and welfare. Our paper differs from theirs in the following important aspects. First, while they only focus on the duopoly downstream market, our model can allow for an arbitrary number of downstream firms and thus can be applied to more general real-world scenarios in which cross-licensing takes place. Second, the way we model R&D behaviours of downstream firms differs from theirs. Unlike their environment in which innovations in the downstream market can raise the quality levels of final products, we focus on cost-reducing process innovation. Third, the timings of the two models also differ. Under their model setup, the upstream manufacturer first needs to decide whether or not to commit to cross-licensing. Then after observing the innovation levels of the downstream firms (in stage three), the manufacturer is able to sign a final contract with the downstream firms. By contrast, we consider cross-licensing and no cross-licensing as two separate strategies of the upstream firm. Finally, some of our results also differ from theirs. For instance, while they find that cross-licensing could improve downstream innovation levels in certain scenarios, we show that cross-licensing will always hinder downstream firms' incentives to innovate in the absence of remedy but will always improve it otherwise. Hence, our finding appears to be more consistent with the Chinese government's decision in the antitrust case of Qualcomm.

Our paper also contributes to the literature on patent licensing and patent pools. Early works study the situations where leading firms may want to license their technologies to their rivals through royalties (Katz and Shapiro, 1985; Kamien and Tauman, 1986). Lerner and Tirole (2004, 2015) discuss how the patent pool is formed and how such innovations should be priced under the patent pool and Standard Setting Organisation. They also assume that firms use royalty fees to maximize pool members' joint profit. In recent work, Reisinger and Tarantino (2019) studied the incentives of vertically integrated firms to form patent pools in the presence of downstream competition.

Moreover, our paper also addresses the complementary problem proposed by Cournot (1838). A similar issue is further studied by Shapiro (2000). We extend this literature by studying vertically related markets in which a dominant firm employs reverse licensing to downstream competitors (Kim, 2004).

Lastly, our paper makes a significant contribution to the ongoing calls for reform in the patent system, with the goal of achieving a more optimal equilibrium between safeguarding intellectual property and promoting fair competition (Polanvyi, 1944; Gallini, 2002; Jaffe, 2000; Boldrin and Levine, 2013). Specifically, we explore the concept of reverse licensing, which is closely intertwined with the notion of a "pass-through right". This revolves around the patent holder granting explicit permission to a third party to license the patented technology, while ensuring that the rights of the patent holder remain intact and unaffected.

The remainder of the paper proceeds as follows. Section 2 introduces the benchmark model and investigates the impacts of upstream reverse licensing on market competition and welfare. Section 3 discusses the implications of the remedy on reverse licensing. Section 4 extends the benchmark model in several directions. Finally, Section 5 concludes. All proofs of propositions and lemmas are relegated in the appendix.

2 Model

We consider a market with an upstream firm U (or manufacturer) that sells an intermediate good to two competing downstream firms D_1 and D_2. The upstream firm is the sole provider of the intermediate good, and the downstream firms will use the intermediate good to produce final products for consumers.[9] For example, Qualcomm owned the crucial patent for 4G technology and thus became a de facto monopoly in the market of 4G smartphone chips. Downstream smartphone firms (such as Huawei and ZTW) needed to incorporate chips in producing a variety of smartphones.

Consumers consider final products as homogeneous in our baseline model.[10] Downstream firms engage in the Cournot competition. For the sake of tractability, we assume the following linear inverse demand function $P = a - Q$, where $a > 0$ and $Q = q_1 + q_2$ is the market demand for the final product, and q_i (i=1,2) is the output chosen by downstream firm i.

For the upstream firm, the marginal cost of producing each unit of the intermediate good is $d_0 \geq 0$. As input price d_0 cannot be more than output price P, we assume that d_0 cannot be too large. For example, it must be less than a. Without loss of generality, the fixed cost is normalized to zero.[11] The manufacturer charges a linear wholesale price d for each unit of the intermediate good sold.[12] Hence, the profit for the manufacturer is $\pi_0 = (d - d_0)Q$.

Assume that for each downstream firm, producing one unit of the final goods requires one unit of the intermediate good (e.g., producing one smartphone requires one chip). Both downstream firms have zero fixed production costs, but they have different (initial) marginal production costs. Suppose that firm 1 is weakly more efficient than firm 2 in production, i.e., $0 < c_1 \leq c_2$. For example, two smartphone producers Huawei and ZTE started with a different number of patents so that they have different production efficiencies. Denote the cost difference by $\Delta c = c_2 - c_1 \geq 0$, which is assumed to be not too large. This ensures that the less efficient firm (firm 2) will always remain active in the market. The cost difference parameter Δc measures the cost asymmetry of the two downstream

[9] We consider an extended model with multiple competing downstream firms in Section 4.8.

[10] We extend the model to allow for differentiated products in Section 4.3.

[11] We also consider a scenario in which the upstream firm can also undertake R&D activities so as to reduce its marginal cost of production in Section 4.6.

[12] One may worry that the manufacturer may set different prices for downstream firms. However, arbitrage severely limits such price discrimination.

firms, which plays a pivotal role in determining firms' behaviours as shown in the following sections. We further denote $s \equiv a - c_1 - d_0 > 0$, which measures the efficient output level ex-ante R&D of downstream firms.

Each firm $i \in \{1, 2\}$ can innovate to reduce its marginal cost by x_i. The cost of innovation for firm i is $\Gamma_i(x_i)$, where $\Gamma_i'(\cdot) > 0$ and $\Gamma_i''(\cdot) > 0$. Without loss of generality, we assume that $\Gamma_i(x_i) = \frac{1}{2}k_i x_i^2$, where $k_i > 0$. The parameter k_i captures the efficiency of firm i's innovation technology: a higher k_i means that it is more costly for firm i to undertake cost-reducing R&D activities. To simplify our algebra, our baseline model focuses on the case where two downstream firms face the same innovation cost, i.e., $k_1 = k_2 = k$. [13]. Note that the limiting case $k \to \infty$ corresponds to the short-run scenario where R&D level of each firm is taken as given. [14] To facilitate our analysis, we make the following two assumptions in the baseline model:

Assumption 1: $k > \frac{4}{3}$ and $\Delta c < \frac{2(3k-4)s}{3(7k-4)}$.

Assumption 2: $c_1 > \frac{2s}{9k-4} + \frac{15k-4}{(9k-4)(3k-4)}$.

The first part of Assumption 1 says that each downstream firm's cost of innovation is not sufficiently low, and the second part of Assumption 1 says that the cost asymmetry ex ant R&D is moderate. Assumption 1 ensures that in equilibrium both downstream firms have some incentives to innovate in all scenarios. Assumption 2 claims that the initial marginal cost of the efficient firm (firm 1) is not too small, which guarantees that firm 1's marginal cost ex-post R&D continues to remain positive. Both Assumptions 1 and 2 together make sure that all equilibrium outcomes are interior and well-defined.

After innovations take place, firms may trade their technology before competition in the product market. There are various scenarios in which trades of technologies occur in the innovation market. In our baseline model, we focus on two extreme scenarios: (1) status quo and (2) reverse licensing.

In the case of status quo, there is no exchange in technologies between downstream firms ex-post R&D. One plausible justification for this scenario is that firms may consider their advanced technologies as trade secrets, which they are reluctant to share with their rivals. Another possible reason is that patent protection might be so weak that it is too costly for each downstream firm to license its new technology to the rival. Therefore, under status quo, the profit for firm i is given by $\pi_i = (P - d - c_i + x_i)q_i - \frac{k_i}{2}x_i^2$.

By contrast, in the case of reverse licensing, there is a free exchange of technologies between

[13] We further discuss the case when $k_1 \neq k_2$ in Section 5.8 and the case when $k_1 < \infty$ and $k_2 \to \infty$ in Section 5.9

[14] It is also important for analysis when there is a technological boundary on R&D. For example, there has been a recent debate about Moore's law that there is a physical limit on how small a chip can be made.

downstream firms. That is, each downstream firm has access to the technology of its rival through a free cross-licensing agreement with the upstream manufacturer. We further assume their innovations are perfectly incompatible across firms. Hence, with access to the combined technologies owned by firms 1 and 2, the marginal cost would become $\min\{c_1 - x_1, c_2 - x_2\}$ for both firms.[15] Then under reverse licensing, the profit for firm i is given by $\pi_i = (P - d - \min\{c_i - x_i, c_j - x_j\})q_i - \frac{k_i}{2}x_i^2$ for $j \neq i$. As compared to the case of status quo, reverse licensing improves the technology of the weak firm but has no direct impact on the technology owned by the strong firm.

To investigate the implications of reverse licensing, we consider a four-stage complete-information dynamic game between the upstream manufacturer and the two downstream consumers. In the first stage, the upstream manufacturer announces the input price d. For example, the chip producer often announces the launch of new product with their price (Popma et al. 2006). In the second stage, both downstream firms undertake R&D. That is, each downstream firm i simultaneously and independently chooses the innovation level x_i. In the third stage, a technology transfer between the two downstream firms may take place depending on whether reverse licensing occurs. In the absence of reverse licensing, no technology transfer takes place.[16] With reverse licensing, both firms will exchange their technology for free.[17] In the last stage, both firms engage in Cournot competition.

Our timing assumption reflects that the dominant firm in the upstream market typically has the capability to affect downstream firms' R&D activities as well as their production decisions.[18] We employ the subgame perfect equilibrium as our solution concept.

3 Equilibrium Analysis and Results

In this section, we first analyze the equilibrium outcomes for cases of status quo and reverse licensing separately, and then study the impacts of reverse licensing on equilibrium outcomes by comparing these two situations.

[15] We consider an alternative setting with compatible technologies in Section 4.2. The qualitative result remains unchanged. Hence, we focus on the current case for the ease of exposition.

[16] We consider the possibility that one downstream firm may license its superior technology to its rival even if reverse licensing does not occur in Section 4.1.

[17] In Section 3, we examine the impact of remedy on reverse licensing by allowing monetary transfers between the two downstream firms once reverse licensing takes place.

[18] In Section 4.5, we consider an alternative timing assumption by allowing downstream firms to undertake R&D before the upstream firm sets the wholesale price. That is, we allow the downstream firms to compete in the Cournot manner immediately after the upstream firm sets prices. The R&D stage is ex-post to the price setting, to avoid more surplus extraction from the upstream firm.

3.1 Status Quo

Under status quo, the equilibrium outcome is fully characterized in the following lemma.

Lemma 1. *When there is no technology transfer between the downstream firms (status quo), in the equilibrium, the upstream firm sets the input price* $d^{no} = d_0 + \frac{s}{2} - \frac{\Delta c}{4}$. *Downstream firms choose the R&D levels* $x_1^{no} = \frac{2s}{9k-4} + \frac{15k-4}{(9k-4)(3k-4)}\Delta c$ *and* $x_2^{no} = \frac{2s}{9k-4} - \frac{3(7k-4)}{(9k-4)(3k-4)}\Delta c$, *and the outputs* $q_1^{no} = \frac{3k}{4}x_1^{no}$ *and* $q_2^{no} = \frac{3k}{4}x_2^{no}$. *Total output is* $Q^{no} = \frac{3k(2s-\Delta c)}{2(9k-4)}$. *The profit of the upstream firm and downstream firms 1 and 2 are* $\pi_0^{no} = \frac{3k(2s-\Delta c)^2}{8(9k-4)}$, $\pi_1^{no} = \frac{(9k-8)k}{16}(x_1^{no})^2$, *and* $\pi_2^{no} = \frac{(9k-8)k}{16}(x_2^{no})^2$.

When there is no technology transfer between two downstream firms, there is no negative externality of R&D and each firm will obtain the full benefit from the cost reduction through innovations. Since the more efficient firm (firm 1) produces a higher final output and obtains a larger profit in the absence of R&D, the marginal benefit of the cost-saving from innovation is always higher. Hence, it has a stronger incentive to innovate than its rival does ($x_1^{no} > x_2^{no}$) when $\Delta c > 0$. With a lower initial cost and a higher R&D level, the more efficient firm would produce more ($q_1^{no} > q_2^{no}$) and obtain a higher profit ($\pi_1^{no} > \pi_2^{no}$).

Lemma 1 further shows that the asymmetry between downstream firms (Δc) plays some role in affecting firms' behaviours. On the one hand, the profit, output and R&D level of firm 1 are all increasing in Δc, while those of firm 2 are all decreasing in Δc. This is natural since the asymmetry between downstream firms tends to strengthen the dominant position of the efficient firm and weaken its rival. On the other hand, it is interesting to see that both the upstream firm's profit and the input price are decreasing in Δc. It reflects that a higher degree of the asymmetry level Δc dampens downstream firms' incentives to innovate and thus lowers their output levels in the product market. As the input demand becomes lower and more elastic, the upstream firm has to lower its input price.

3.2 Reverse Licensing

Now we consider the situation where a royalty-free cross-licensing agreement is imposed by the upstream firm. For instance, the upstream firm is only willing to sell its intermediate good to downstream firms conditional on the "unconditional surrender" of their existing technologies.

Unlike the previous case with no reverse licensing, once downstream firms have surrendered all (incompatible) technologies to the upstream firm, only the best technology will be adopted by both firms in the process of production. Hence, reverse licensing may hinder the innovation incentives of downstream firms (e.g., the less efficient downstream firm), and then change the pricing strategy of the upstream firm in the first place. In particular, when the technology is fully incompatible, only one firm would innovate in equilibrium. As in an anti-coordination game, there would be two possible equilibria–one that only firm 1 innovates and the other that only firm 2 innovates. Since the upstream firm has higher profit under the former equilibrium, we will focus on the equilibrium that firm 1 only innovates. [19] The following lemma summarizes the equilibrium outcome under reverse licensing.

Lemma 2. *Under reverse licensing, in equilibrium, the upstream firm sets the input price* $d^r = d_0 + \frac{s}{2}$. *For R&D innovation, firm 1 invests to reduce their marginal costs by* $x_1^r = \frac{s}{9k-2}$ *and firm 2 invests zero. In the product market, downstream firms produce* $q_1^r = q_2^r = \frac{1}{2}Q^r = \frac{3sk}{2(9k-2)} = \frac{3}{2}kx_1^r$. *The equilibrium profit of the upstream firm and downstream firms 1 and 2 are* $\pi_0^r = \frac{3ks^2}{2(9k-2)}$, $\pi_1^r = \frac{s}{4}kx_1^r = \frac{s^2k}{4(9k-2)}$, *and* $\pi_2^r = (q_2^r)^2 = \frac{9s^2k^2}{4(9k-2)^2}$.

Under reverse licensing, since only the superior technology will be used by the upstream firm, there exists a negative externality of innovations between two downstream firms: only one firm will want to do R&D and free ride on the rival's R&D activities. It is natural that only the more efficient firm (firm 1) will do R&D in equilibrium. The reason is that given $c_1 < c_2$, it always requires firm 2 to spend more to appreciate its technology to a superior level than its rival does, i.e., $c_2 - x_2 < c_1 - x_1$ implies that $x_2 > x_1$ and $\frac{k(x_2)^2}{2} > \frac{k(x_1)^2}{2}$. As a result, both downstream firms compete with the same technology and produce the same level of final outputs. However, the more efficient firm (firm 1) obtains a lower profit than its rival due to its expenditure in R&D.

Another observation is that the upstream manufacturer sets the same input price for all downstream producers, despite that firm 1 already possessed the technology. The intuition is that the upstream manufacturer would extract more surplus from a more competitive downstream market. This accounts for Qualcomm's conduct in China, where it treated Huawei, ZTE and Xiaomi all equally even though the former two owned much more patents than Xiaomi back in 2015.

3.3 Comparison

Using Lemmas 1 and 2, we can evaluate the impact of reverse licensing based on the status quo. We are interested in how reverse licensing affects the wholesale price, R&D levels, total output (or

[19] Since the upstream firm has monopoly power, it is natural to expect that the monopoly can induce the more desired equilibrium. For example, the upstream firm can simply offer the reverse licensing to firm 1 only. Moreover, when technology is partly compatible, both firms would innovate and firm 1 would innovate more than firm 2. Hence, the equilibrium that only firm 2 innovates does not satisfy the continuity in terms of compatibility of technology.

consumer surplus) and firm's profits.

3.3.1 Effect on Wholesale Price

First, comparing the input prices in both cases, we always have $d^r - d^{no} = \frac{\Delta c}{4} > 0$. Hence, the input price is always higher under reverse licensing. This follows naturally as the innovation market is less competitive because firm 2 has no incentive to do R&D under reverse licensing. From the perspective of the upstream firm, the product demand is less sensitive to the change of input price under reverse licensing. The following proposition formally summarizes the result.

Proposition 1. *Reverse licensing will always yield a higher wholesale price.*

3.3.2 Effect on R&D

Second, we would like to see how the innovation market is affected by reverse licensing. First, since the less efficient firm (firm 2) can free-ride on firm 1's R&D activities, reverse licensing fully eliminates firm 2's R&D incentives (i.e., $x_2^{no} > 0 = x_2^r$). Second, due to the existence of negative externalities, the efficient firm (firm 1) would also have a lower incentive to innovate under reverse licensing in the absence of any effect on the input price. However, an inflated input price (by Proposition 1) would further hinder firm 1's R&D incentive. The following proposition formally summarizes these results.

Proposition 2. *Reverse licensing will always yield a lower R&D expenditure by each firm, and hence lead to a lower total R&D level.*

3.3.3 Effect on Total Output, Price and Consumer Surplus

Third, we investigate the impact of reverse licensing on product market competition. Since the total output is negatively correlated to the effective marginal costs of production for both downstream firms, it would be useful to study the impacts of reverse licensing on them. Define the effective marginal cost of firm 1 under reverse licensing by $\tilde{c}_1^r = c_1 - x_1^r + d^r$ and under status quo by $\tilde{c}_1^{no} = c_1 - x_1^{no} + d^{no}$. Likewise, the effective marginal cost of firm 2 under reverse licensing by $\tilde{c}_2^r = \tilde{c}_1^r$ and under status quo by $\tilde{c}_2^{no} = c_2 - x_2^{no} + d^{no}$. The results are summarized in the following corollary.

Corollary 1. *Reverse licensing always raises the effective marginal cost for firm 1, and it lowers the effective marginal cost for firm 2 if* $\Delta c > \frac{12(3k-4)s}{(9k-2)(27k-20)}$.[20]

[20]It is easy to verify that $\frac{12(3k-4)s}{(9k-2)(27k-20)} < \frac{2(3k-4)s}{3(7k-4)}$ is always satisfied whenever $k > 4/3$.

The results in Corollary 1 can be understood using Propositions 1 and 2. First, the impact of reverse licensing on firm 1's effective marginal cost is

$$\Delta c_1 = \tilde{c}_1^r - \tilde{c}_1^{no} = \underbrace{(d^r - d^{no})}_{(+)} + \underbrace{(x_1^{no} - x_1^r)}_{(+)}.$$

It is clear that firm 1's effective marginal cost is always higher under reverse licensing ($\tilde{c}_1^r > \tilde{c}_1^{no}$) given a higher input price ($d^r > d^{no}$) and a lower R&D level ($x_1^r < x_1^{no}$). However, firm 2's effective marginal cost might be lower because of the diffusion of technology from its rival. In fact, the impact of reverse licensing on firm 2's effective marginal cost is

$$\Delta c_2 = \tilde{c}_2^r - \tilde{c}_2^{no} = \underbrace{(d^r - d^{no})}_{(+)} + \underbrace{(x_2^{no} - x_2^r)}_{(+)} \underbrace{-(\Delta c + x_1^r - x_2^r)}_{(-)}.$$

Hence, reverse licensing lowers firm 2's effective marginal cost if $\Delta c > x_2^{no} - x_1^r + d^r - d^{no}$, which occurs when Δc is sufficiently high.

In particular, the impact of reverse licensing on downstream firms' effective marginal costs of production can be explained by three effects. First, reverse licensing raises the wholesale price charged by the upstream firm, which is further passed on to downstream firms in the production market. Hence, it tends to raise the final price and lower allocative efficiency (**pricing effect**). Second, reverse licensing also weakens the R&D incentives of both downstream firms due to the negative externality in the innovation market (**innovative effect**). Third, reverse licensing can improve productive efficiency by ensuring the diffusion of the most efficient technology in the production market (**competitive effect**). Moreover, the first and third effects are related to static efficiency, while the second one corresponds to dynamic efficiency. As a result, reverse licensing always lowers dynamic efficiency but may improve static efficiency.

We further explore how these three effects work for both downstream firms. The following table summarizes these effects. The first two effects are present for firm 1, which results in a higher effective marginal cost under reverse licensing. For firm 2, since all three effects are present, the net effect on its effective marginal cost would depend on whether the third positive effect dominates the other two. Specifically, the pricing effect for firm 2 is captured by the term $d^r - d^{no} = \frac{\Delta c}{4}$, which is positively related to Δc. The innovative effect is $x_2^{no} - x_2^r = \frac{2s}{9k-4} - \frac{3(7k-4)}{(9k-4)(3k-4)}\Delta c$ for firm 2, which is negative related to Δc. The competitive effect is $\Delta c + x_1^r - x_2^r = \Delta c + \frac{s}{9k-2}$, which is also positively related to Δc. Hence, the competitive effect strictly dominates the other two when Δc is large (or when the initial cost asymmetry between downstream firms is sufficiently high), in which case reverse licensing lowers the effective marginal cost of firm 2.

Table 1: Three effects on effective marginal costs of downstream firms

	Pricing effect	Innovative effect	Competitive effect
Firm 1	$\frac{\Delta c}{4}$	$\frac{9ks}{(9k-4)(9k-2)} + \frac{15k-4}{(9k-4)(3k-4)}\Delta c$	no
Firm 2	$\frac{\Delta c}{4}$	$\frac{2s}{9k-4} - \frac{3(7k-4)}{(9k-4)(3k-4)}\Delta c$	$\Delta c + \frac{s}{9k-2}$
Firms 1&2	$\frac{\Delta c}{2}$	$\frac{(27k-4)s}{(9k-4)(9k-2)} - \frac{2}{9k-4}\Delta c$	$\Delta c + \frac{s}{9k-2}$

Corollary 1 is very useful for analyzing the impact of reverse licensing on the total output, which is negatively related to the effective marginal costs of production for both downstream firms. Moreover, since the final price is negatively related to and consumer surplus is positively related to the total output, we have the following result.

Proposition 3. *Reverse licensing results in a higher total output, a lower price and a higher consumer surplus only when* $k > k^{CS} = \frac{8}{3}$ *and* $\Delta c > \Delta c^{CS} = \frac{4s}{9k-2}$.

Proposition 3 shows that reverse licensing can result in a lower price and improve consumer surplus when the cost of R&D investments is sufficiently high and two downstream firms are sufficiently asymmetric ex-ante innovation. The results can be explained by the three effects which we identified before. Note that the total output in the production market is written as $Q = \frac{2a-\tilde{c}_1-\tilde{c}_2}{3}$, which is negatively correlated to the aggregation of effective marginal costs of two downstream firms. From Corollary 1, while reverse licensing always raises the effective marginal cost of firm 1, $\tilde{c}_1$, its impact on the effective marginal cost of firm 2, $\tilde{c}_2$, is generally ambiguous. The net effect of reverse licensing on the total effective marginal cost of two downstream firms can only be negative when the competitive effect on firm 2 overwhelms two other negative effects on both downstream firms.

We can decompose these effects using Table 1. On the one hand, a higher Δc tends to strengthen the positive competitive effect and the negative pricing effect, but weaken the negative innovative effect. Since the former two effects always dominate, the net effect will result in a lower aggregate effective marginal cost. On the other hand, a higher k tends to weaken both the innovative effect and the competitive effect, and it has no impact on the pricing effect. The net effect would also result in a lower aggregate marginal cost. As a result, when both Δc and k are sufficiently high, i.e., when the cost of R&D investments is sufficiently high and two downstream firms are sufficiently asymmetric ex-ante innovation, the net effect of reverse licensing on the effective aggregate marginal cost is negative, which results in a higher total output and a higher consumer surplus. Otherwise, the final price is higher and consumers are worse off under reverse licensing.

3.3.4 Effect on Firms' Profits

Fourth, we investigate how reverse licensing affects firms' profitability. The impact of reverse licensing on the upstream firm's profit is summarized as follows.

Proposition 4. *Reverse licensing raises the upstream firm's profit when* $k > k^U = \frac{4(12+\sqrt{53})}{39} \approx 1.98$ *and* $\Delta c > \Delta c^U = 2(1 - \sqrt{\frac{9k-4}{9k-2}})s$*.*

Proposition 4 shows that the upstream firm can benefit from reverse licensing when the cost of R&D investments is sufficiently high and the initial cost asymmetry between downstream firms is also sufficiently high. The intuition is as follows. Unlike Proposition 3, the upstream firm's profit is not directly affected by the pricing effect since the wholesale price is fully controlled by the upstream firm itself. By contrast, the upstream firm cares about the total output generated in the downstream market, which is jointly determined by the other two effects (innovative effect and competitive effect): while the innovative effect lowers the total output, the competitive effect can improve it. Hence, the net effect on the total output is positive when the positive competitive effect dominates. This occurs when the cost of R&D investments is sufficiently high (the innovative effect is weak) and the initial cost asymmetry between downstream firms is sufficiently high (the competitive effect is strong).

Finally, the effects of reverse licensing on the profits of downstream firms are summarized in the following proposition.

Proposition 5. *Reverse licensing always raises the profit of firm 2, and it also raises the profit of firm 1 if* $\Delta c < \Delta c_1$*, where* $\Delta c_1 = \frac{2(3k-4)}{15k-4}\left(\frac{9k-4}{\sqrt{(9k-2)(9k-8)}} - 1\right)s$*.*

The above proposition shows that two downstream firms can also benefit from reverse licensing. First, firm 2 always benefits from reverse licensing. This reflects that reverse licensing not only enables firm 2 to save costs in R&D investments but also ensures the full diffusion of advanced technology from its rival. In other words, the innovative effect and competitive effect are both positive for firm 2. As a result, these two positive effects always dominate the negative effect of inflated wholesale price. By contrast, firm 1 may not always benefit from reverse licensing. This naturally follows from Lemmas 1 and 2: compared to its rival, firm 1 earns a lower profit under reverse licensing, and a higher profit under status quo. Intuitively, while the innovative effect tends to be positive (since reverse licensing reduces firm 1's excessive R&D investments), the effect of competitive effect is always negative (since reverse licensing weakens firm 1's market dominance by reducing its rival's cost of production). Hence, firm 1 can only benefit from reverse licensing when the initial cost asymmetry of downstream firms is sufficiently small. In this case, the negative competitive effect is weak enough so that the positive innovative effect always dominates the other two.

3.3.5 Effect on Total Welfare

The effect of reverse licensing on total welfare is the aggregate effects of reverse licensing on consumers and firms. Recall that when $k^U < k < k^{CS}$, we always have $\Delta c^U > \Delta c_1$; when $k > k^{CS}$, we have $\Delta c_1 < \Delta c^U < \Delta c^{CS}$. We summarize these effects for different parameter value k in the following tables.

Table 2: Case 1: $k < k^U$

	$\Delta c < \Delta c_1$	$\Delta c > \Delta c_1$
Firm 2	+	+
Firm 1	+	−
U	−	−
CS	−	−

Table 3: Case 2: $k^U < k < k^{CS}$

	$\Delta c < \Delta c_1$	$\Delta c_1 < \Delta c < \Delta c^U$	$\Delta c > \Delta c^U$
Firm 2	+	+	+
Firm 1	+	−	−
U	−	−	+
CS	−	−	−

Table 4: Case 3: $k > k^{CS}$

	$\Delta c < \Delta c_1$	$\Delta c_1 < \Delta c < \Delta c^U$	$\Delta c^U < \Delta c < \Delta c^{CS}$	$\Delta c > \Delta c^{CS}$
Firm 2	+	+	+	+
Firm 1	+	−	−	−
U	−	−	+	+
CS	−	−	−	+

We formally summarize the results as follows.

Proposition 6. *Reverse licensing will improve total welfare if (i)* $\underline{k} < k < \overline{k}$ *and* $\underline{\Delta c} < \Delta c < \overline{\Delta c}$*; or (ii)* $k > \overline{k}$ *and* $\Delta c > \underline{\Delta c}$*, where* $\underline{k} \approx 2.81$ *and* $\overline{k} \approx 3.12$*, while* $\underline{\Delta c}$ *and* $\overline{\Delta c}$ *are the cutoff values of* Δc

which depend on the parameter k.[21]

Note that we always have $k^U < k^{CS} < \underline{k} < \overline{k}$. It implies that when $k < k^{CS}$ so that reverse licensing always lowers consumer surplus, it also reduces total welfare as well. When $\underline{k} < k < \overline{k}$, it is possible that reverse licensing will improve welfare, which occurs when the initial cost asymmetry between downstream firms is in the medium range. Finally, when $k > \overline{k}$, reverse licensing will improve total welfare only when Δc is not too small, which reflects that the positive effects on the upstream firm, firm 2 and consumers always dominate the negative effect on firm 1.

3.3.6 Profitable Reverse Licensing and Welfare Effects

We have so far analyzed the effects of reverse licensing on all parties. In this section, we focus on the case where $k > k^U$ and $\Delta c > \Delta c^U$, which ensures that the upstream firm strictly prefers reverse licensing to status quo. This also corresponds to the situation where the upstream firm would want to impose reverse licensing if it is able to decide whether or not to adopt it. The following proposition summarizes the impacts of reverse licensing on downstream firms, consumer surplus and welfare.

Proposition 7. *Suppose $k > k^U$ and $\Delta c > \Delta c^U$, then the upstream firm benefits from reverse licensing. In this case, reverse licensing will*

- *always benefit firm 2 but hurt firm 1;*
- *increases output, reduces output price and improve consumer surplus when $k > k^{CS}$ and $\Delta c > \Delta c^{CS}$;*
- *improve total welfare only when (i) $\underline{k} < k < \tilde{k}$ and $\underline{\Delta c} < \Delta c < \overline{\Delta c}$; or when (ii) $\tilde{k} < k < \overline{k}$ and $\Delta c < \overline{\Delta c}$, or when (iii) $k > \overline{k}$, where $\tilde{k} \approx 3.08$.*

Our findings in Proposition 7 and in other previous propositions offer several key insights. First, we characterize the conditions under which a dominant upstream firm is willing to adopt reverse licensing so as to restrict the way in which downstream firms undertake innovations. The practice of reverse licensing is profitable from the perspective of the upstream firm only when the cost of R&D innovations is sufficiently high and the initial cost asymmetry between downstream firms is also sufficiently high.

[21] The detailed expressions for these cutoffs are relegated in the appendix.

Second, whenever the upstream firm prefers to impose reverse licensing, reverse licensing will unambiguously hurt the efficient downstream firm (firm 1) but benefit the less efficient one (firm 2). The efficient downstream firm is worse off due to the free diffusion of its R&D investments despite the cost savings in R&D innovations. The less efficient firm is better off since it not only saves the R&D costs but also obtains the advanced technology from its rival at no cost.

Third, the effects of reverse licensing on consumer surplus and welfare are in general ambiguous, which depend on the cost of R&D innovations and the initial cost asymmetry between downstream firms. A necessary condition that reverse licensing is beneficial to both consumer surplus and total welfare is when both the cost of R&D innovations and the initial cost asymmetry between downstream firms are sufficiently high. Furthermore, it is possible that reverse licensing changes consumer surplus and total welfare in the opposite direction. For instance, when k is in the middle range (i.e., $k^{CS} < k < \overline{k}$) and Δc is sufficiently high, reverse licensing will improve consumer surplus but lower total welfare. By contrast, when $k > \underline{k} \approx 2.81$ and Δc is in the middle range, reverse licensing will lower consumer surplus but improve total welfare. The above findings suggest that we should be cautious towards the evaluations of reverse licensing, which in general rely on antitrust policies.

The above key insights can be used to explain the practice of reverse licensing in the Qualcomm antitrust case and resolve some antitrust concerns. Two key features of the smartphone market in China can be identified. First, the competition is rather intensive and the innovations of smartphone companies are generally costly. Second, the market structure is generally asymmetric and only a few smartphones capture a majority of market shares. From our theory (Proposition 4), the dominant firm in the upstream market (Qualcomm) can indeed benefit from imposing reverse licensing on the downstream markets. Smartphone firms in the downstream markets generally go against the practice of reverse licensing, which can be explained by the fact that more efficient downstream firms are in general worse off under reverse licensing (Proposition 5). Even though reverse licensing can potentially be beneficial to consumers and the whole society, this can only arise when the cost of R&D in the smartphone market is extremely high and the asymmetry among smartphone companies is rather strong (Proposition 7). These, however, do not seem to fit the reality. Hence, it is more natural that reverse licensing is anti-competitive. Therefore, to promote the local research environment, it is crucial for the authorities to fine Qualcomm for such a practice. The following section will discuss the case of remedy, where the upstream firm has to compensate the downstream firms for the technology surrendered.

Finally, our theory also highlights the importance of considering the R&D incentives of downstream firms when evaluating the possible effects of reverse licensing. Suppose we only restrict our discussion to a static model without modelling firms' R&D incentives, which corresponds to the case where $k \to \infty$. Then we find that reverse licensing will always improve the profit of the upstream firm as well as consumer surplus.[22] That is, in the absence of the innovative effect, the overall effect of reverse licensing on the total output and consumer surplus is always positive. However, such a pro-competitive effect of reverse licensing can be reversed once the innovative effect has been taken into account. This suggests that we should not provide policy implications for reverse licensing simply based on static models in which innovative incentives (such as R&D) are absent.

4 Remedy

Our previous section highlights the fact that profitable reverse licensing for the upstream firm may hinder R&D innovations in the downstream market and lower consumer surplus or total welfare. A natural solution to solve such inefficiency is to adopt some remedy for the upstream firm. One possible way is to require that the upstream firm should pay for the technologies purchased in the downstream market. For instance, under the ruling of the Chinese government in 2013, Qualcomm has been ordered to compensate downstream firms for the cross-licensing agreement. This implies that Qualcomm cannot exercise the market power to force downstream producers to surrender their patents without offsetting royalties.

To evaluate the possible effects of such a remedy, we revise the previous game under reverse licensing by allowing the upstream firm to offer royalty compensations to downstream firms in the third stage (i.e., the licensing stage). More specifically, we assume that in the second stage in which downstream firms conduct R&D, the upstream firm makes take-it-or-leave-it offers to purchase the technologies from the downstream firms with per unit offsetting royalty fees r_1 and r_2, respectively, and downstream firms simultaneously decide whether or not to accept the offer.[23]. Downstream firms keep and use their own technologies if they reject the offer. In other words, status quo is the outside option for them in the equilibrium computation.

When downstream firm i accepts the offer, the effective input price for i is $d - r_i$. If both firms

[22] Note that when $k \to \infty$, the total output is $Q^{no} = \frac{1}{3}(s - \frac{1}{2}\Delta c)$ and the profit of the upstream firm is $\pi_0^{no} = \frac{(2s-\Delta c)^2}{24}$ under status quo. Under reverse licensing, the total output is $Q^r = \frac{1}{3}s$ and the profit of the upstream firm is $\pi_0^r = \frac{s^2}{6}$. Hence, we always have $d^{no} < d^r$ and $Q^{no} < Q^r$.

[23] Royalty fee licensing is often modelled as take-it-leave-it offers from the innovators (Kaimen 1992)

accept the offers, the effective marginal cost for i is $d - r_i + \min\{c_i - x_i, c_j - x_j\}$. If firm i accepts the offer but not for firm j, then the effective marginal costs for i and j are $d - r_i + c_i - x_i$ and $d + c_j - x_j$. If both firms reject the offer, the situation would be exactly as in the status quo.

The following lemma summarizes the equilibrium outcome.

Lemma 3. *Under remedy, in equilibrium, the upstream firm sets the wholesale price* $d^{re} = d_0 + \frac{1}{2}s + \frac{\Delta c}{4} + \frac{3k}{2(3k-2)(9k-4)}s + \frac{3}{9k-4}\Delta c$. *Downstream firms produce* $q_1^{re} = \frac{3k(2s(3k-1)+3(3k-2)\Delta c)}{4(9k-4)(3k-2)}$ *and* $q_2^{re} = \frac{2s(3k-1)-(9k-2)\Delta c}{4(9k-4)}$. *Total output is* $Q^{re} = q_1^{re} + q_2^{re} = \frac{2s(3k-1)^2+(3k-2)\Delta c}{2(9k-4)(3k-2)}$. *For R&D innovation, firm 1 invests to reduce their marginal costs by* $x_1^{re} = \frac{2s(3k-1)+3(3k-2)\Delta c}{(9k-4)(3k-2)}$ *and firm 2 invests zero. The offsetting royalty fees are* $r_1^{re} = \frac{2(3k-1)s+(9k-1)(3k-2)\Delta c}{2(9k-4)(3k-2)}$ *and* $r_2^{re} = 0$. *The profit of the upstream firm is* $\pi_0^{re} = \frac{4(3k-1)^2 s^2 + 4s(3k-2)\Delta c - (9k+2)(3k-2)(\Delta c)^2}{8(9k-4)(3k-2)}$. *The profits for downstream firms 1 and 2 are* $\pi_1^{re} = \frac{k(9k-8)}{16}(x_1^{re})^2$ *and* $\pi_2^{re} = (q_2^{re})^2$.

The above lemma shows that under remedy only the efficient downstream firm would want to innovate as in the case of reverse licensing. However, the upstream firm will now compensate the innovator (firm 1) by paying a positive royalty fee ($r_1^{re} > 0$). Comparing Lemmas 2 and 3, we can further explore the impacts of the remedy, the result of which is summarized as follows.

Proposition 8. *Compared to reverse licensing, a remedy will*

- *yield a higher wholesale price for the intermediary input, higher total R&D expenditure, lower output price, more output, and higher consumer surplus;*
- *increase the profit for the upstream firm when* $\Delta c < \Delta c^{re}$, *where* $\Delta c^{re} = \frac{2s}{9k+2}\left(1 + \sqrt{\frac{6k(2+3k)(-4+9k)}{4-24k+27k^2}}\right)$[24]*;*
- *increase the efficient downstream firm's profit, and decrease the inefficient downstream firm's profit.*

Since the remedy allows the downstream innovator to get partially compensated for its R&D expenditure, it can potentially improve the R&D incentive of firm 1. This further enables the upstream firm to charge a higher intermediary price. The effective marginal costs of firms 1 and 2 under remedy are $\tilde{c}_1^{re} = c_1 - x_1^{re} + d^{re} - r_1^{re}$ and $\tilde{c}_2^{re} = c_1 - x_1^{re} + d^{re}$. Hence, the impact of the remedy on firm 1's effective marginal cost is $\Delta c_1 = \tilde{c}_1^{re} - \tilde{c}_1^{r} = (d^{re} - d^{r}) - (x_1^{re} - x_1^{r}) - r_1^{re} < 0$. That is, the joint effects of the remedy by raising the R&D level and the royalty fee always dominate the effect of raising the input price, so firm 1's effective marginal cost always becomes lower. Similarly, the impact of

[24] Under Assumptions 1 and 2, $\Delta c < \Delta c^{re}$ is always satisfied when $k < k^{re} \approx 6.804$.

the remedy on firm 2's effective marginal cost is $\Delta c_2 = \tilde{c}_2^{re} - \tilde{c}_2^{r} = (d^{re} - d^{r}) - (x_1^{re} - x_1^{r})$, which is also negative when $\Delta c > \frac{6(-2+3k)s}{(-10+9k)(-2+9k)}$. Since the aggregate effect on both firms' costs is always negative (i.e., $\Delta c_1 + \Delta c_2 < 0$), the remedy always leads to a higher output level and hence improves consumer surplus. Interestingly, the upstream firm's profit could be better even under remedy when the cost asymmetry between the two downstream firms is weak. The intuition is that with the efficient technology transferred, firm 2 could produce more. The profit gain from firm 2's *extra production* would outweigh the royalty compensation to firm 1, yielding a higher net profit for the upstream firm. Finally, firm 1 is always better off under remedy since it gets partially compensated for its R&D expenditure, while firm 2 is worse off because of its weaker competitiveness in the production market.

The above findings show that the remedy on reverse licensing will improve consumer surplus, and hence provide theoretical support for the ruling of the Chinese government on Qualcomm's antitrust case by forcing Qualcomm to compensate smartphone companies. Besides, when the cost asymmetry between downstream firms is not too large, the remedy also tends to raise the profits of the upstream firm and the efficient downstream firm, so it will be a Pareto improvement.

5 Extensions

In this section, we consider several extensions to our baseline framework. We either show the robustness of our main findings or derive additional new insights. The detailed analyses (or proofs) of these extensions are relegated in the online appendix.

5.1 Independent Royalty Licensing

In the baseline model, we have assumed that in the absence of reverse licensing (i.e., status quo), downstream firms will utilize their own new technologies independently. We now consider a situation where a downstream firm with a superior technology ex-post R&D may want to license its technology voluntarily to its rival so as to extract more surplus through licensing fees. In particular, we consider an alternative status quo scenario where in the R&D stage, the downstream firm with a superior technology (or a lower marginal cost) can license its technology to the other firm through royalty licensing.

While the equilibrium outcome under reverse licensing remains the same as in the baseline setting, the possibility of voluntary licensing in the downstream market changes the equilibrium outcome in status quo. Nevertheless, we find that the key results in the baseline model continue to hold. For instance, reverse licensing will also raise the wholesale price and lower R&D levels of downstream firms as compared to the status quo. Moreover, reverse licensing could lead to higher consumer surplus and upstream firm's profit, when the initial cost of production for the downstream firms is asymmetric. It is also possible that reverse licensing is profitable for the upstream firm, but it lowers consumer surplus and/or total welfare. The intuition can also be explained by the three effects identified in the baseline setting.

We further explore the possibility of downstream firms' voluntary licensing on the impacts of reverse licensing. On the one hand, voluntary licensing in the downstream market raises the licensor's profit ex-post R&D, which increases or subdues the firm's R&D incentives. This result is fairly standard in the literature of technology licensing in horizontal markets (e.g., Chang et. al, 2023). On the other hand, the change of R&D incentives in the downstream market will affect the nature of competition and thus affect the way in which the upstream firm sets wholesale prices. As a result, as compared to the baseline model with no licensing, voluntary licensing of downstream firms may generate ambiguous effects on the profits of firms, consumer surplus and welfare under status quo. In particular, when the cost of R&D investments is sufficiently high (i.e., k is large enough), voluntary licensing tends to lower the upstream firm's profit and consumer surplus, making reverse licensing more beneficial to both the upstream firm and consumer surplus.

5.2 Compatible Technology

For the ease of analysis, we have so far focused on the simple case with incompatible technologies. To show that our key results do not crucially rely on this particular type of technology, in this section, we consider a more general case where the innovations from both firms can be (partially) compatible under reverse licensing. To model the compatibility of technologies, we assume that the marginal cost of production for firm i remains $c_i - x_i$ under status quo, and it becomes $c_i - x_i - \alpha x_j$ under reverse licensing, where $0 \leq \alpha \leq 1$ is the parameter that measures the degree of compatibility of different technologies: a higher α means that two technologies of downstream firms are more compatible. At extreme, if $\alpha = 0$, then the two technologies are fully incompatible. If $\alpha = 1$, the two technologies are fully compatible.

Since the compatibility of technologies does not affect firms' costs of production under status quo, the equilibrium outcome under status quo remains the same (Lemma 1). By contrast, the equilibrium outcome of reverse licensing is different from Lemma 2, which naturally depends on the degree of compatibility α. We can show that except that the input price is independent of α, outputs and R&D levels by downstream firms, profits for the upstream firm and downstream firms are all increasing in α. Hence, compared to status quo, reverse licensing will continue to lead to a higher intermediary input price and lower total R&D expenditures, regardless of the degree of compatibility. In this sense, the compatibility of technologies does not qualitatively change the price effect and innovative effect identified in the baseline model. In addition, since firms' profits and consumer surplus are all increasing in α, it follows that as the technologies become more compatible, the upstream firm finds it more profitable to adopt reverse licensing. In the meantime, consumers are also more likely to be better off. It implies that reverse licensing tends to raise less concern in markets where firms compete with more compatible technologies.

5.3 Technology Spillovers between Downstream Firms

One key nature of R&D is the spillover effect that firms generate for others. We have abstracted away this feature so far. In this section, we extend the baseline model by allowing for a general degree of technology spillovers between downstream firms. Suppose the marginal cost of firm i after innovation is $\tilde{c}_i = c_i - x_i - \beta x_j$, where $\beta \in [0, 1]$ captures the magnitude of R&D spillovers. A higher β means that the spillover effect is stronger or the patent protection is weaker. There is no spillover when $\beta = 0$ (baseline setting), and full spillover when $\beta = 1$. This case is also analogous to the situation where the innovation from both firms is compatible. Some innovation (i.e. βx_2) from firm 2 will be complimentary to firm 1's innovation.

Compared with no licensing case, it is not surprising that reverse licensing will always yield a higher retail price for the intermediary input regardless of β.

For consumer surplus, it is similar to the previous comparison. Higher downstream asymmetry ($\underline{\Delta c}^{SP} < \Delta c < \overline{\Delta c}^{SP}$) and higher research cost ($k > k^{SP}$) will lead to higher consumer surplus for reserve licensing. The intuition also follows the previous discussion. But there are interesting observations regarding the cutoff k^{SP} and $\overline{\Delta c}^{SP} - \underline{\Delta c}^{SP}$. $k_{SP} = \frac{1}{3}(8 - 2\beta - 6\beta^2)$ is decreasing in β when $\beta < \frac{1}{4}(\sqrt{21} - 1)$, and it also decreases in β when $\beta > \frac{1}{4}(\sqrt{21} - 1)$. Hence, a higher β lowers the cutoff k^{SP}, which tends to make reverse licensing more beneficial to consumer surplus. On the other hand, $\overline{\Delta c}^{SP} - \underline{\Delta c}^{SP}$ is always increasing in β. Hence, a higher β tends to make reverse licensing more beneficial to consumer surplus, since the range under which reverse licensing improves consumer surplus becomes larger. In sum, a higher spillover makes reverse licensing more beneficial to consumer surplus. It is intuitive that higher spillover will create more innovation, which, then, reduces the cost of production, leading to higher total output.

For R&D expenditure, it is now possible that reverse licensing leads to higher R&D levels for both firms. For x_1: when there is no spillover, reverse licensing always results in lower x_1. A similar result also holds when the spillover is small ($\beta < \frac{1}{6}(\sqrt{22}+1)$. When the spillover is large, reverse licensing can lead to higher x_1 if k is small and Δc is large. The intuition is that firm 1 would like to maintain its leading position through more R&D. [25] For x_2: when there is no spillover, reverse licensing always results in lower x_2 (since $x_2 = 0$ under reverse licensing). With spillover, x_2 is always positive under reverse licensing. And reverse licensing always results in lower x_2 if Δc is small; otherwise, x_2 will be higher under reverse licensing. The intuition is that regardless of the level of spillover, if the initial asymmetry between the two firms is large, firm 2 would like to narrow the gap through more R&D.

For the upstream firm's profit, it is similar to the 0 spillover case, where reverse licensing leads to higher profit when research cost is high and the cost difference between downstream firms is high. We can also observe that when spillover is high, reverse licensing will lead to a higher upstream firm's profit. The intuition follows from the consumer surplus argument, since the upstream firm's profit is determined by the intermediary input price (which is always higher under reverse licensing) and the total output level.

5.4 Differentiated Products

Our baseline model focuses on the case with homogeneous products. It has been shown that there is a complex relationship between competition innovation (e.g., Aghion et al., 2005; Tang, 2006; Marshall and Parra, 2019). To show that this assumption is not critical to our key findings, we explore the situation with differentiated products in this section. Suppose the inverse demand function for firm i takes the following linear form $P_i = a - q_i - bq_j$, for $i \neq j$, and $b \in [0, 1]$. The parameter b measures the degree of substitutability of downstream firms' products. A higher b means that the two products offered by firms are closer substitutes. In particular, products become homogeneous when $b = 1$.[26]

We first show that the impacts of reverse licensing on the wholesale price and R&D levels of downstream firms are similar to those in the baseline model, which do not rely on the degree of

[25] Under the compatibility analogy, when firm 2's technology is more complementary, firm 1 would like to maintain its leading position by doing more research.

[26] The inverse demand is derived from the following quadratic utility function $U(q_1, q_2) = a(q_1+q_2) - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 - bq_1q_2$. The corresponding consumer surplus can also be written as $CS = U(q_1, q_2) - P_1q_1 - P_2q_2 = \frac{1}{2}(q_1^2 + q_2^2 + 2bq_1q_2)$.

product differentiation. Unlike the case of homogeneous products, consumer surplus will no longer merely rely on the total output of downstream firms but is closely related to the individual output of each downstream firm. However, our results continue to show that reverse licensing will yield a higher consumer surplus if the initial cost asymmetry of downstream firms is sufficiently high and the cost of R&D is not too low ($k > \bar{k}^{PD}$). We find a similar result for the upstream firm's profit. Hence, production differentiation does not change the key mechanisms of the baseline model essentially.

We further explore the effect of product differentiation on the potential impacts of reverse licensing. On the one hand, reverse licensing is more likely to be beneficial for consumer surplus when the degree of production differentiation is low (i.e., $b < \bar{b}^{PD}$) as compared to the case where $b > \bar{b}^{PD}$. In addition, since $\frac{d\bar{k}^{PD}}{db} > 0$, a higher degree of differentiation b makes reverse licensing less likely to be beneficial for consumer surplus. Likewise, we also achieve a similar result for the upstream firm's profit. It can also be checked that when the goods are closer substitutes (i.e., b is higher), reverse licensing is less likely to be profitable for the upstream firm. These results can be explained by the fact that a higher degree of production differentiation tends to dissipate the firms' profits in the downstream market, which tends to weaken the beneficial competitive effect.

5.5 Alternative Timing of the Game

In this section, we relax our timing assumption by considering an alternative game structure. In particular, we allow downstream firms to conduct R&D and surrender the technologies to the upstream firm before the latter firm sets the input price. This timing assumption may reflect the fact that firms' pricing and production behaviours are easier to adjust than R&D activities, which typically take a longer duration and may suffer from a high risk of failure. In this scenario, the upstream firm may want to adjust its wholesale price according to the R&D levels of downstream firms. Unlike the baseline setting, the wholesale price will no longer affect downstream firms' incentives to innovate.

The new timeline of the game is as follows. In the first stage, downstream firms simultaneously undertake R&D activities. In the second stage, downstream firms may cross-license their technologies (if any), depending on whether reverse licensing takes place. In the third stage, the upstream firm sets the intermediary input price d. In the last stage, both downstream firms engage in Cournot competition.

Under this new setting, we derive two interesting observations. First, reverse licensing will lead to a lower price for the intermediary input. In this way, the R&D outputs serve as a royalty "remedy" to the downstream firms. Second, when the research cost is sufficiently high (i.e., $k > \frac{5(a-c_1)+\Delta c}{18\Delta c}$), both the upstream firm's profit and consumer surplus will be improved. Note that $\frac{dk}{d\Delta c} < 0$, when the cost asymmetry is high, it is easier for reverse licensing to yield better outcomes. The intuition is that licensing will intensify the downstream competition, as it closes the gap between firm 1 and firm 2. Thus, total output will be higher (the upstream firm's profit and consumer surplus are both increasing in total output).

5.6 Upstream Innovation

We have so far assumed that the cost for the upstream firm is exogenously fixed so that reverse licensing only affects R&D incentives of downstream firms but not those of the upstream firm. Since reverse licensing can be beneficial to the upstream firm, it is natural that it may also improve the R&D incentive of the upstream firm. In this section, we consider such a possibility by allowing the upstream firm to endogenously determine its technology.

Consider the case where the upstream firm is also able to conduct R&D. To be consistent with the baseline model, we assume that the initial marginal cost of producing the intermediary input is d_0 for the upstream firm. The upstream firm can further reduce its marginal cost by x_0 if it invests $\frac{1}{2}\kappa x_0^2$, where $\kappa > 0$ measures the cost of innovation for the upstream firm. We further revise the model in the baseline setting by adding one more stage (say stage 0) before stage 1, in which the upstream firm chooses the R&D level x_0.

Our findings show that the key results in the baseline model remain largely intact. In particular, the initial cost asymmetry of downstream firms and the cost of R&D continue to play a vital role in determining the impact of reverse licensing. Reverse licensing tends to be beneficial for the upstream firm, consumer surplus and welfare when both the initial cost asymmetry and the cost of R&D of downstream firms are sufficiently high. Moreover, we also show that reverse licensing tends to cause an additional beneficial dynamic efficiency effect by improving the R&D incentives of the upstream firm. This new effect will lower the effective marginal cost of production for the upstream firm, which may result in a lower wholesale price under reverse licensing. In other words, the positive dynamic efficiency effect in the upstream market may reverse the negative pricing effect caused by reverse licensing. Therefore, as compared to the case where the R&D incentive of the upstream firm is absent, reverse licensing tends to be more beneficial for consumer surplus and welfare as it weakens the pricing effect and innovative effect **(i.e. less negative)**.

5.7 Multiple Downstream Firms

In the baseline model, we focus on the simple structure of the downstream market by assuming that there are only two downstream firms. However, it has been shown that technology transfer among two competitors is very different from markets with more than two competitors in horizontal markets (Creane et al., 2013).

In this section, we relax this assumption by allowing for an arbitrary number $n \geq 3$ of downstream firms. To simplify the analysis, we focus on the case with one superior downstream firm and all other symmetric firms. Specifically, we assume that firms 2 to n are symmetric. That is, $c_1 < c_2 = c_3 = \ldots = c_n = c$. As in the baseline model, we continue to assume that $k_1 = \ldots = k_n = k$. Our model with more than two downstream firms delivers tractable results, and thus contributes to the existing literature on cross-licensing in vertically related markets (e.g., Wang et al., 2023).

The main findings are as follows. First, reverse licensing continues to result in a higher input price regardless of the number of downstream firms n because $d^r - d^{no} = \frac{n-1}{2n}\Delta c > 0$. In particular, since the gap in input price difference is increasing in n, reverse licensing tends to generate a higher price effect by raising the input price as the number of downstream firms increases. Second, reverse licensing also hinders firms' incentives to innovate by lowering the R&D levels for each downstream firm. Third, while reverse licensing continues to raise the upstream firm's profit and consumer surplus when the cost asymmetry among downstream firms is strong and the cost of R&D investment is high, such positive effects will be stronger as n increases. Finally, the impacts of reverse licensing on downstream firms' profit are in general ambiguous, which depend on all parameters Δc, k and n. However, when the cost asymmetry among firms Δc is sufficiently small, we find that reverse licensing always benefits all downstream firms regardless of n. The above results deliver a key message about the impacts of the downstream market structure on reverse licensing: as the downstream market becomes more competitive (e.g., n is larger), reverse licensing tends to generate a stronger price effect (which favours the upstream firm), and stronger competition effect (which is in favour of consumer surplus). Hence, the antitrust concern for reverse licensing should also be lessened in industries with less concentration and more competing firms in the downstream markets.

5.8 Asymmetric R&D efficiency

We have so far assumed that the cost of R&D is symmetric among downstream firms. This enables us to deliver clean results and focus more on analyzing the role of the cost asymmetry between downstream firms. In this section, we formally model the different R&D efficiencies of downstream firms by allowing for $k_2 = k_1 + \Delta k$ with $\Delta k > 0$. It reflects that the less efficient downstream firm may also incur a higher cost of R&D to reduce its cost of production.

Our results continue to show that reverse licensing always leads to a higher input price and a lower R&D level of each downstream firm, which are the same as in the baseline model. From the perspective of firms' profits, the research efficiency plays a similar role as the cost efficiency for the upstream firm and the efficient downstream firm (firm 1). However, it affects the profit of firm 2 differently. The reason is that the cost efficiency gap may largely hinder firm 2's incentives to innovate under status quo especially when k_2 is large enough. Hence, reverse licensing causes a stronger free-riding effect, which benefits firm 2 more. Besides, the impacts of reverse licensing on consumer surplus and total welfare are analogous to those in the baseline model.

5.9 Only One Downstream Innovator

Consider the case of n downstream firms. Firm 1's marginal cost of production is c, while the other $n - 1$ firms face a higher cost $c + \Delta c$. Suppose only firm 1 is able to innovate, or in other words, the innovation cost for the other $n - 1$ firms are very high, i.e. $k_2 \to \infty$.

Our results remain similar. That is, reverse licensing always lowers the firm's incentive to innovate and increases the per-unit fee charged by the upstream firm. It could improve the profit of the upstream firm if the cost of innovation (for firm 1) is sufficiently high and the firms' cost asymmetry is moderate. Under the same condition, it will lower the total output and consumer surplus in most cases.

When firms are ex-ante asymmetric, e.g., one firm is the dominant firm (say Huawei) and all others are small firms. Only the dominant firm is able to undertake R&D activities. It is possible that the upstream firm benefits from using reverse licensing, but reverse licensing is anti-competitive (e.g., lowers consumer surplus). Intuitively, reverse licensing creates a key trade-off between static efficiency and dynamic efficiency. On the one hand, reverse licensing ensures the diffusion of advanced technology which improves the competitive effect. This intensifies downstream competition which benefits both the upstream firm and consumers. On the other hand, reverse licensing hinders the innovator's incentive to innovate. The overall effect on firms' profit and consumer surplus will then depend on which of the two effects dominates.

6 Conclusion

In this paper, we develop a theoretical model to analyze the business practice of reverse licensing in vertically related markets, i.e., downstream firms surrender their R&D outputs to the upstream firm and, subsequently, the technologies are licensed back to the downstream firms. We find that such a practice could be beneficial for a dominant firm in the upstream market under plausible conditions. Our paper thus provides a rationale for why dominant firms in the markets (such as Qualcomm) may engage in such potentially anti-competitive practices.

We further investigate the welfare impacts of reverse licensing. Our findings uncover three important effects that reverse licensing can generate on the market competition: while reverse licensing always causes a pricing effect and an innovative effect which are detrimental to efficiency, it can result in a beneficial competitive effect by ensuring the diffusion of advanced technology in the downstream market. We show that the positive competitive effect will dominate only when the initial cost asymmetry and the cost R&D of downstream firms are both sufficiently high, in which case reverse licensing will improve consumer surplus and welfare.

Our model also predicts that profitable reverse licensing (which improves the upstream firm's profit) can be anti-competitive in the sense that it lowers consumer surplus and/or welfare under certain circumstances. This key finding not only rationalizes the practice of reverse licensing from dominant firms in vertically related markets (e.g., Qualcomm), but also confirms the serious concern for such a practice raised by antitrust authorities. Hence, from the perspective of antitrust regulation, reverse licensing should be treated under the rule of reason instead of per se illegal. We further suggest that a royalty remedy that obliges the dominant upstream firm to adequately compensate downstream innovators can largely solve this problem. Such a remedy may not only raise downstream firms' incentives to innovate, but also improve both the upstream firm's profit and consumer surplus. Our theory can also be applied to the influential antitrust case of Qualcomm, and also be utilized to explain the ruling of the Qualcomm case in China.

Our main findings also shed light on the classical complementary inputs problem by Cournot (1838). Shapiro (2000) further asserts that if the inputs are technologies, patent pool can avoid this problem in horizontal markets. We have extended this literature by suggesting that cross-licensing imposed by dominant upstream firms on downstream firms in the vertically related market could also avoid such a problem.

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A Appendix: Proofs of Propositions and Lemmas

This appendix collects all proofs of propositions and lemmas.

A.1 Proof of Lemma 1

By backward induction, in the last stage of Cournot competition, firm D_1 chooses q_1 to maximize $\pi_1 = (a - q_1 - q_2 - (c_1 - x_1 + d))q_1 - \frac{k}{2}x_1^2$ and firm D_2 chooses q_2 to maximize $\pi_2 = (a - q_1 - q_2 - (c_2 - x_2 + d))q_2 - \frac{k}{2}x_2^2$. The equilibrium outputs are given by $q_1 = \frac{1}{3}(a - 2c_1 + c_2 - d + 2x_1 - x_2)$ and $q_2 = \frac{1}{3}(a + c_1 - 2c_2 - d - x_1 + 2x_2)$.

In stage 2, firm D_1 chooses x_1 to maximize $\pi_1 = \frac{1}{9}(a - 2c_1 + c_2 - d + 2x_1 - x_2)^2 - \frac{k}{2}x_1^2$ and firm D_2 chooses x_2 to maximize $\pi_2 = \frac{1}{9}(a + c_1 - 2c_2 - d - x_1 + 2x_2)^2 - \frac{k}{2}x_2^2$. Hence, D_1 will choose $x_1(d) = \frac{4((3k-4)(a-d)) + c_1(4-6k) + 3c_2k)}{(16-48k+27k^2)}$ and D_2 will choose $x_2(d) = \frac{4((3k-4)(a-d)) + c_2(4-6k) + 3c_1k)}{(16-48k+27k^2)}$. The corresponding outputs are $q_1(d) = \frac{3k}{4}x_1(d) = \frac{3k((3k-4)(a-d)) + c_1(4-6k) + 3c_2k)}{(16-48k+27k^2)}$ and $q_2(d) = \frac{3k}{4}x_2(d) = \frac{3k((3k-4)(a-d)) + c_2(4-6k) + 3c_1k)}{(16-48k+27k^2)}$. The total output is $Q(d) = \frac{3k(2(a-d) - c_1 - c_2)}{9k-4}$.

In stage 1, firm U chooses d to maximize $\pi_U = (d - d_0)Q(d)$. The equilibrium wholesale price is $d^{no} = \frac{1}{4}(2a - c_1 - c_2 - 2d_0) = d_0 + \frac{s}{2} - \frac{\Delta c}{4}$, where $s = a - c_1 - d_0$.

Therefore, we can further obtain that the equilibrium R&D levels are $x_1^{no} = \frac{2s}{9k-4} + (\frac{2}{3k-4} - \frac{1}{9k-4})\Delta c$ and $x_2^{no} = \frac{2s}{9k-4} - (\frac{2}{3k-4} + \frac{1}{9k-4})\Delta c$. The equilibrium outputs for both downstream firms are $q_1^{no} = \frac{3k}{4}x_1^{no}$ and $q_2^{no} = \frac{3k}{4}x_2^{no}$. The equilibrium total output is $Q^{no} = \frac{3k(2s-\Delta c)}{2(9k-4)}$. The equilibrium firms' profits are $\pi_o^{no} = \frac{3k(2s-\Delta c)^2}{8(9k-4)}$, $\pi_1^{no} = \frac{(9k-8)k}{16}(x_1^{no})^2$ and $\pi_2^{no} = \frac{(9k-8)k}{16}(x_2^{no})^2$.

To ensure the existence of the equilibrium, we require that $k > \frac{4}{3}$ and $x_2^{no} > 0$, i.e., $k > \frac{4}{3}$ and $\Delta c < \frac{2s(3k-4)}{3(7k-4)}$.

A.2 Proof of Lemma 2

By backward induction, in the last stage of Cournot competition, firm D_1 chooses q_1 to maximize $\pi_1 = (a - q_1 - q_2 - (c_1 - x_1 + d))q_1 - \frac{k}{2}x_1^2$ and firm D_2 chooses q_2 to maximize $\pi_2 = (a - q_1 - q_2 - (c_1 - x_1 + d))q_1$. The equilibrium outputs are given by $q_1 = q_2 = \frac{1}{3}(a - c_1 - d + x_1)$.

In stage 2, since only firm D_1 is doing R&D, it chooses x_1 to maximize $\pi_1 = \frac{1}{9}(a - c_1 - d + x_1)^2 - \frac{k}{2}x_1^2$. Then the optimal R&D level is $x_1(d) = \frac{2(a-c_1-d)}{9k-2}$. The corresponding outputs are $q_1(d) = q_2(d) = \frac{3k(a-c_1-d)}{9k-2}$, and the total output is $Q(d) = \frac{6k(a-c_1-d)}{9k-2}$.

In stage 1, firm U chooses d to maximize $\pi_0 = (d - d_0)Q(d)$. Hence, the equilibrium wholesale price is $d^r = \frac{a-c_1+d_0}{2} = d_0 + \frac{s}{2}$.

As a result, firm D_1 chooses $x_1^r = \frac{s}{9k-2}$ and firm D_2 invests zero. Firms' outputs are $q_1^r = q_2^r = \frac{1}{2}Q^r = \frac{3}{2}kx_1^r$. Firms' equilibrium profits are $\pi_o^r = \frac{3ks^2}{2(9k-2)}$, $\pi_1^r = \frac{s^2k}{4(9k-2)}$ and $\pi_2^r = \frac{9s^2k^2}{4(9k-2)^2}$.

To ensure the existence of the equilibrium, we only require that $k > \frac{2}{9}$.

A.3 Proof of Proposition 3

Comparing the total outputs in both scenarios, we have $Q^r - Q^{no} = -\frac{3k(4s-(9k-2)\Delta c)}{2(9k-2)(9k-4)}$. Given that $k > \frac{4}{3}$, we have $Q^r > Q^{no}$ if and only if $4s < (9k-2)\Delta c$. Moreover, given $\Delta c < \frac{2(3k-4)s}{3(7k-4)}$, it can be verified that $\frac{4s}{9k-2} < \frac{2(3k-4)s}{3(7k-4)}$ is satisfied only when $k > 8/3$. Hence, $Q^r > Q^{no}$ when $k > 8/3$ and $\frac{4s}{9k-2} < \Delta c$.

A.4 Proof of Proposition 4

Comparing the profits of the upstream firm in both scenarios, we have $\pi_0^r - \pi_0^{no} = \frac{3k(-(9k-2)(\Delta c)^2+4(9k-2)s\Delta c-8s^2)}{8(9k-2)(9k-4)}$. Hence, we have $\pi_0^r > \pi_0^{no}$ if and only if $(2+2\sqrt{\frac{9k-4}{9k-2}})s > \Delta c > (2-2\sqrt{\frac{9k-4}{9k-2}})s$.

Given that $k > 4/3$ and $\Delta c < \frac{2(3k-4)s}{3(7k-4)}$, we know that $2(1+\sqrt{\frac{9k-4}{9k-2}})s > \frac{2(3k-4)s}{3(7k-4)}$ is always satisfied, and $\frac{2(3k-4)s}{3(7k-4)} > 2(1-\sqrt{\frac{9k-4}{9k-2}})s$ is only satisfied when $k > \frac{4(12+\sqrt{53})}{39}$. Hence, the upstream firm will only benefit from reverse licensing when $k > \frac{4(12+\sqrt{53})}{39}$ and $\Delta c > 2(1-\sqrt{\frac{9k-4}{9k-2}})s$.

A.5 Proof of Proposition 5

First, comparing the profits of firm D_1, we have

$$\begin{aligned}\Delta\pi_1(\Delta c) &= \pi_1^r - \pi_1^{no} \\ &= \frac{k}{16}\left(\frac{4s^2}{9k-2} - \frac{(9k-8)(2(3k-4)s+(15k-4)\Delta c)^2}{(3k-4)^2(9k-4)^2}\right).\end{aligned}$$

Given $k > \frac{4}{3}$, it is clear that $\Delta\pi_1(\Delta c)$ is strictly decreasing in Δc, since $\frac{d\Delta\pi_1}{d\Delta c} = -\frac{k(15k-4)(9k-8)(2(3k-4)s+(15k-4)\Delta c)}{8(3k-4)^2(9k-4)^2} < 0$. Moreover, we have $\Delta\pi_1(0) = \frac{9k^2s^2}{2(9k-4)^2(9k-2)} > 0$. Hence, there is a unique Δc, denoted by $\Delta c_1 > 0$, such that $\Delta\pi_1 > 0$ if and only if $\Delta c < \Delta c_1$. Note that Δc_1 is given by

$$\Delta c_1 = \frac{2(3k-4)}{15k-4}\left(\frac{9k-4}{\sqrt{(9k-2)(9k-8)}} - 1\right)s.$$

It can be verified that $\Delta c_1 < \frac{2(3k-4)s}{3(7k-4)}$.

Second, comparing the profits of firm D_2, we have

$$\begin{aligned}\Delta\pi_2(\Delta c) &= \pi_2^r - \pi_2^{no} \\ &= \frac{k}{16}\left(\frac{36ks^2}{(9k-2)^2} - \frac{(9k-8)(2(3k-4)s-3(7k-4)\Delta c)^2}{(3k-4)^2(9k-4)^2}\right)\end{aligned}$$

We have $\frac{d\Delta\pi_2}{d\Delta c} = \frac{3k(7k-4)(9k-8)(2(3k-4)s-3(7k-4)\Delta c)}{8(3k-4)^2(9k-4)^2} > 0$ given $\Delta c < \frac{2(3k-4)s}{3(7k-4)}$. Moreover, we have $\Delta\pi_2(0) = \frac{k(8-45k+81k^2)s^2}{(9k-4)^2(9k-2)^2} > 0$. Therefore, we always have $\Delta\pi_2(\Delta c) > 0$.

A.6 Proof of Proposition 6

Under status quo, the total welfare is

$$W^{no} = (Q^{no})^2/2 + \pi_0^{no} + \pi_1^{no} + \pi_2^{no} = f(\Delta c) = \frac{k((208-552k+369k^2)\Delta c^2 - 20(3k-4)^2 s\Delta c + 20(3k-4)^2 s^2)}{8(3k-4)^2(9k-4)}.$$

Under reverse licensing, the total welfare is

$$W^r = (Q^r)^2/2 + \pi_0^r + \pi_1^r + \pi_2^r = \frac{k(45k-7)s^2}{2(9k-2)^2}.$$

It is clear that W^{no} (or $f(\Delta c)$) is decreasing in Δc when $\Delta c < \frac{10(3k-4)^2 s}{208-552k+369k^2}$, and increasing in Δc otherwise. Note that when $f(0) = \frac{5ks^2}{2(9k-2)}$, and $f(\frac{10(3k-4)^2 s}{208-552k+369k^2}) = \frac{10k(9k-8)s^2}{208-552k+369k^2}$. Moreover, W^r does not depend on Δc. It is easy to verify that $f(0) > W^r$ is always satisfied for any $k > 4/3$, and $f(\frac{10(3k-4)^2 s}{208-552k+369k^2}) > W^r$ if and only if $k < \underline{k}$, where $\underline{k}$ is the third root to the equation $-272+2248k-2661k^2+675k^3 = 0$ and $\underline{k} \approx 2.81$. In addition, we have $f(\frac{2(3k-4)s}{3(7k-4)}) = \frac{4k(-14+27k)s^2}{9(-4+7k)^2}$ and $f(\frac{2(3k-4)s}{3(7k-4)}) > W^r$ if and only if $k < \overline{k}$, where $\overline{k}$ is the third root to the equation $-560+5112k-8919k^2+2349k^3 = 0$ and $\overline{k} \approx 3.12$.

Therefore, given $k > 4/3$ and $\Delta c < \frac{2(3k-4)s}{3(7k-4)}$, we have the following results: if $k < \underline{k}$, we have $W^{no} > W^r$ always holds; if $\underline{k} < k < \overline{k}$, we have $W^{no} < W^r$ when $\underline{\Delta c} < \Delta c < \overline{\Delta c}$, and $W^{no} > W^r$ otherwise; if $k > \overline{k}$, we have $W^{no} < W^r$ when $\underline{\Delta c} < \Delta c$ and $W^{no} > W^r$ otherwise. Note that $\underline{\Delta c}$ and $\overline{\Delta c}$ are the smaller and the larger root of the quadratic equation $f(\Delta c) = W^r$, where

$$\begin{aligned}\underline{\Delta c} &= \frac{2(3k-4)^2}{208-552k+369k^2}\left(5 - \frac{\sqrt{3(-272+2248k-2661k^2+675k^3)}}{(3k-4)(9k-2)}\right)s, \\ \overline{\Delta c} &= \frac{2(3k-4)^2}{208-552k+369k^2}\left(5 + \frac{\sqrt{3(-272+2248k-2661k^2+675k^3)}}{(3k-4)(9k-2)}\right)s.\end{aligned}$$

A.7 Proof of Proposition 7

First, given $k > k^U$ and $\Delta c > \Delta c^U$, it is easy to verify that $\pi_1^r < \pi_1^{no}$ and $\pi_2^r > \pi_2^{no}$ always hold. Second, we can also show that $Q^r > Q^n$ if and only if $k > k^{CS}$ and $\Delta c > \Delta c^{CS}$. Since $P = a - Q$, we have the result for $P^r < P^n$. Last, comparing W^r with W^n, we find that $W^r > W^n$ if (i) $\underline{k} < k < \tilde{k}$ and $\underline{\Delta c} < \Delta c < \overline{\Delta c}$, or when (ii) $\tilde{k} < k < \overline{k}$ and $\Delta c < \overline{\Delta c}$, or when (iii) $k > \overline{k}$, where $\tilde{k}$ is the third root to the equation $-55552 + 564480k - 1998432k^2 + 2888784k^3 - 1715337k^4 + 314928k^5 = 0$ and $\tilde{k} \approx 3.08$.

A.8 Proof of Lemma 3

By backward induction, in the last stage of Cournot competition if reverse licensing happens, the equilibrium quantities chosen by two firms are $q_i^* = \frac{a-2\tilde{c}_i+\tilde{c}_j}{3}$ where $\tilde{c}_i = \min\{c_i - x_i, c_j - x_j\} + d - r_i$. Hence, $Q^* = q_1^* + q_2^* = \frac{a-\tilde{c}_1-\tilde{c}_2}{3}$ As $P(Q) = a - \frac{a-2\tilde{c}_1+\tilde{c}_2}{3} - \frac{a-2\tilde{c}_2+\tilde{c}_1}{3} = \frac{a-\tilde{c}_1-\tilde{c}_2}{3}$, we have $\pi_i^* = \left(\frac{a-2\tilde{c}_i+\tilde{c}_j}{3}\right)^2 - \frac{k}{2}x_i^2$. If there is no cross-licensing, it would be the same except that $\tilde{c}_i = c_i - x_i$.

Next, consider stage 3 where the upstream firm chooses an offsetting royalty fee to acquire innovations from downstream firms to incorporate into the chips. Formally, we have

$$\max_{r_1,r_2} \pi_0 = (d - d_0)Q - r_1 q_1 - r_2 q_2$$

As in reverse licensing, we only focus on the subgame where only firm 1 innovates. It is because that innovations are perfect substitutes between firms. The offsetting royalties for firm 2 would be zero if firm 1 has accepted the offer from the upstream manufacturer. Then firm 1's optimal R&D investment would be independent of the choice of firm 2. Hence, firm 2 would have no incentive to invest but to free ride. With $x_2 = 0$, we have $r_2 = 0$ and the optimization becomes $\max_{r_1} dQ - r_1 q_1$.

Since downstream firms are not forced to surrender technology, the upstream firms needs to make sure that firm 1 is willing to accept the agreement than to keep the technology. Under the proposed scheme, we have $\tilde{c}_1 = c_1 - x_1 + d - r_1$ and $\tilde{c}_2 = c_1 - x_1 + d$. When firm 1 keeps her technology, we have $\tilde{c}_1 = c_1 - x_1 + d$ and $\tilde{c}_2 = c_2 + d$ instead. Hence, the individual rationality constraint for firm 1 is

$$\frac{(a - 2(c_1 - x_1 + d - r_1) + (c_1 - x_1 + d))^2}{9} - \frac{k_1}{2}x_1^2 \geq \frac{(a - 2(c_1 - x_1 + d) + (c_2 + d))^2}{9} - \frac{k_1}{2}x_1^2.$$

Since $\frac{d\pi_0}{dr_1} = -\frac{1}{3}(a - c_1 - 2d + d_0 + 4r_1 + x_1) < 0$, the upstream wants r_1 as low as possible. Then this individual rational constraint must be binding so that $r_1 = \frac{1}{2}(c_2 - c_1 + x_1)$.

Note that this is similar to the Nash bargaining solution. We have $q_1^*(x_1, d) = \frac{a-2(c_1-x_1+d)+(c_2+d)}{3}$ and $q_2(x_1, d) = \frac{a-(c_1-x_1+d)+a-(c_2+d)}{6}$.

Note that we need to make sure that the upstream would have an incentive to impose reverse licensing. That is,

$$(d - d_0)\frac{a - (c_1 - x_1 + d - r_1) - (c_1 - x_1 + d)}{3} \geq (d - d_0)\frac{a - (c_1 - x_1 + d) - (c_2 + d)}{3}$$

which is equivalent to $r_1 \geq c_1 - c_2 - x_1$ and this always hold.

Now consider stage 2. As argued above, we only need to focus on the case that only firm 1 would do R&D. Solving $\max_{x_1} \pi_1$ gives us $x_1^*(d) = \frac{4(a-2c_1+c_2-d)}{9k_1-8}$. This implies that $q_1 = \frac{3(a-2c_1+c_2-d)k_1}{9k_1-8}$, $q_2 = \frac{2(3k_1-2)(a-d)-3k_1c_1-(3k_2-4)c_2}{2(9k_1-8)}$, and $r_1^*(d) = \frac{1}{2}\left(c_2 - c_1 + \frac{4(a-2c_1+c_2-d)}{9k_1-8}\right)$.

Finally for stage 1, the upstream firm solves $\max_d \pi_0 = dQ^*(d) - r_1^*(d)\, q_1^*(d)$. The solution is given by the statement in the text.

A.9 Proof of Proposition 8

First, we have $d^{re} - d^r = \frac{\Delta c}{4} + \frac{3k}{2(3k-2)(9k-4)}s + \frac{3}{9k-4}\Delta c > 0$. Second, $x_1^{re} = \frac{s}{9k-4} + \frac{3s+3(3k-2)}{2(9k-4)(3k-2)}\Delta c > \frac{s}{9k-2} = x_1^r$. Since $x_2^r = x_2^{re} = 0$, the total R&D expenditure is also higher. Third, $Q^{re} - Q^r = \frac{2s(18k^2-3k-2)+(9k-2)(3k-2)\Delta c}{2(9k-2)(3k-2)(9k-4)} > 0$. Hence, since $P = a - Q$, we have $P^{re} < P^r$. Fourth, comparing the upstream firm's profit, we have $\pi_0^{re} - \pi_0^r = \frac{4(18k^2-3k-2)s^2+4(9k-2)(3k-2)s\Delta c-(3k-2)(9k-2)(9k+2)(\Delta c)^2}{8(9k-2)(9k-4)(3k-2)}$, which is positive if $\Delta c < \Delta c^{re} = \frac{2s}{9k+2}\left(1 + \sqrt{\frac{6k(2+3k)(-4+9k)}{4-24k+27k^2}}\right)$. Fifth, it can be checked that $\pi_1^{re} > \pi_1^r$ always holds. Finally, since $q_2^{re} < \frac{2s(3k-1)}{4(9k-4)} < \frac{3ks}{2(9k-2)} = q_2^r$, we always have $\pi_2^{re} < \pi_2^r$.