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1. Introduction

As the Covid-19 shock started to unfold and the concomitant control measures snatched away the livelihood of many, the need for assessing the potential impact without data on different segments of an economy heightened. Unless it is a forecasting exercise, existing methodologies are basically backward looking because they require continuous time series of data before and after the shock to obtain the shock-related estimates. Policymakers cannot wait till data become available to steer the economy away from potential abysmal outcomes. This exercise presents a forward-looking methodology that can be used at the outset of an event like Covid-19 to generate potential growth effects on different sectors of an economy under different scenarios that may manifest. Abeysinghe and Tan (2020a, 2020b) used this methodology in their sectoral analysis of Singapore and Hong Kong.

It is worth stating briefly what the intervention analysis is before moving to the general methodology. A common practice in regression analysis is to use a binary dummy variable to account for a break resulting from interventions like wars, strikes, disease outbreaks, many other crises, new policies or even changes in data compilation methods. In the regression $y_t = \alpha + \delta D_t + \beta' x_t + \varepsilon_t$, D_t is the dummy or the intervention variable that represents the Covid-19 shock in the present context and x_t is a vector of other variables, and ε_t is white noise. If the impact of the shock is confined to the time unit $t*$, then a pulse dummy, $D_t = 1$ if $t = t*$ and 0 otherwise, is used in the regression. If the impact lasts for several periods from $t*$, for example at $t*$, $t*+1$, $t*+2$, $t*+3$, then a step dummy that takes value 1 over the affected periods and 0 otherwise is used. Alternatively, several pulse dummies could be used to account for differentiated impacts during the relevant periods. This type of very familiar intervention analysis, however, does not account for lingering effects that a shock may exert on y_t over a prolonged period.

To account for such lingering effects Box and Tiao (1975) introduced the intervention analysis within the framework of transfer functions. Without any x_t variables on the RHS, the intervention model is of the form:

$$y_t = \frac{\lambda(L)}{\varphi(L)} D_t + u_t \tag{1}$$

where L is the lag operator, $\lambda(L) = \lambda_0 + \lambda_1 L + ... + \lambda_s L^s$, $\varphi(L) = 1 - \varphi_1 L - ... - \varphi_r L^r$ ($\varphi(L)$ with roots outside the unit circle), and $u_t = [\theta(L)/\phi(L)]\varepsilon_t$ is an ARMA process. If the lag orders s=3 and r=1, model (1) gives a geometric decay of the intervention effect after 3 lags. Depending on the values of s and r different impulse response patterns emerge.

Specification of (1) requires some experience and effort especially when x variables also appear on the RHS. A simpler alternative is the Autoregressive Distributed Lag (ARDL) formulation:

$$\varphi(L) y_t = \lambda(L) D_t + v_t \tag{2}$$

where $v_t = \varphi(L) u_t$. Empirically this ARDL specification requires adding sufficient number of lags of y_t to make v_t a white noise process. This may happen at the expense of parsimony that (1) offers. But (2) is more amenable when x variables need to be included in the regression.

One major problem with intervention modelling is that estimation of the model requires the availability of sufficient data after the intervention occurs. Typically, the intervention analysis is carried out in a historical context to understand the dynamics involved and to learn some lessons from the episode. This is the backward-looking approach mentioned above. What is needed is a forward-looking methodology that can provide warning lights to policy makers before the data become available. One objective of this exercise is to present such a methodology for intervention analysis. The other major problem is that models like (1) and (2) provide estimates of the direct impact of the intervention. When a large number of interdependent variables are involved, as with different sectors of an economy, it is important to account for indirect effects as well. The indirect effect is propagated through the other sectors of the economy; when visitor arrivals drop to zero it is not only the transport, accommodation, and food services sectors that suffer but other sectors also suffer depending on the strength of inter-sectoral linkages. The second objective of the methodology is to generalize model (2) for a vector of interdependent variables to derive both direct and indirect effects.

Next section presents the general methodology and Section 3 presents the empirical methodology to be followed. In the absence of data to assess the impact of the shock the methodology requires calibrating the parameters related to the shock. The basic methodology involves combining pre-crisis estimates with calibrated estimates to assess the direct and indirect growth effects of the shock.

2. General Methodology

The standard workhorse when a vector of interdependent variables are involved is the vector autoregression (VAR) framework.[1] As is well known, however, the standard VAR models become unwieldy when the number of variables to be modelled increases. This problem is addressed in various ways in Structural VAR models. The methodology presented here is adapted from Abeysinghe (2001), Abeysinghe and Forbes (2005) and Shen and Abeysinghe (2020). This section presents the general methodology that can be applied to settings similar to the Covid-19 outbreak. The empirical methodology used in the next three chapters is presented in the next section.

Let y_{it} be the growth rate (%) of value added (Y_{it}) of sector *i*. The following equation can be estimated for each sector separately using pre-crisis data.

$$y_{it} = \phi_{0i} + \sum_{j=1}^{p} \phi_{ji} y_{it-j} + \sum_{j=0}^{p} \beta_{ji} y^{*}_{it-j} + \lambda' Z_t + \varepsilon_{it} \tag{3}$$

where $y^{*}_{it} = \sum_{j=1}^{n-1} w_{ijt} y_{jt},\ j \neq i$ is the weighted sum of the growth rate of the remaining sectors. The weights can be worked out in different ways as discussed in the next section. *Z* are other relevant exogenous (control) variables for the sector. The equation can be estimated by OLS, but there is an endogeneity problem because of contemporaneous y^{*}_{it} on the RHS of (3). This is unlikely to be a serious problem as observed in Abeysinghe and Forbes (2005) and Shen and Abeysinghe (2020) where they have tried both OLS and 2SLS.

[1] McKibbin and Fernando (2020) and Maliszewska, Matto and Mensbrugghe (2020) have used the CGE framework to assess the global growth impact of COVID-19 outbreak.

After estimating all equations using pre-crisis data, each y_{it}^{*} can be opened up with estimated βs and weights . Ignoring Z variables and if n=3 and p=1 equation (3) for sector 1 can be expanded as:

$$y_{1t} = \phi_0 + \phi_{11} y_{1t-1} + \beta_{01}(w_{12t} y_{2t} + w_{13t} y_{3t}) + \beta_{11}(w_{12t-1} y_{2t-1} + w_{13t-1} y_{3t-1}) + \varepsilon_{it} \qquad (4)$$

In matrix notation the three equations can be written (without the constant term) as

$$\begin{pmatrix} 1 & -\beta_{01} & -\beta_{01} \\ -\beta_{02} & 1 & -\beta_{02} \\ -\beta_{03} & -\beta_{03} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & w_{12t} & w_{13t} \\ w_{21t} & 1 & w_{23t} \\ w_{31t} & w_{32t} & 1 \end{pmatrix} \begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{pmatrix} =$$

$$\begin{pmatrix} \phi_{11} & \beta_{11} & \beta_{11} \\ \beta_{12} & \phi_{22} & \beta_{12} \\ \beta_{13} & \beta_{13} & \phi_{33} \end{pmatrix} \cdot \begin{pmatrix} 1 & w_{12t-1} & w_{13t-1} \\ w_{21t-1} & 1 & w_{23t-1} \\ w_{31t-1} & w_{32t-1} & 1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{2t} \end{pmatrix} \qquad (5)$$

where the notation "$\cdot$" indicates the Hadamard product giving the element-wise product of two matrices.

Pre-crisis parameter estimates need to be combined with calibrated parameter values for the COVID-19 effect. COVID-19 is represented by the intervention dummy variable D. The full SVAR model in matrix notation for the n sectors can be written as

$$(B_0 \cdot W_t) y_t = \phi_0 + (B_1 \cdot W_{t-1}) y_{t-1} + \ldots + (B_p \cdot W_{t-p}) y_{t-p} + \Gamma_0 D_t + \Gamma_1 D_{t-1} + \ldots + \Gamma_p D_{t-p} + \varepsilon_t \qquad (6)$$

where B are restricted parameter matrices (estimated from pre-crisis data), Γ are diagonal calibrated parameter matrices, and W_t are smoothly changing weights.

Using the lag operator L and by fixing W_t at a desired time point, in shorthand notation $B^w(L) = (B_0 \cdot W) - (B_1 \cdot W)L - \ldots - (B_p \cdot W)L^p$ and $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \ldots + \Gamma_p L^p$, (6) can be written as

$$B^w(L) y_t = \phi_0 + \Gamma(L) D_t + \varepsilon_t \qquad (7)$$

or $$y_t = \phi_0^* + B^w(L)^{-1} \Gamma(L) D_t + u_t \,. \qquad (8)$$

The required impulse responses or growth effects with respect to $D_t = 1$ are given by the matrices $R(L) = B^w(L)^{-1} \Gamma(L)$.

Note that the model parameters are estimated using smoothly changing W_t values and as a result the effective parameter matrices $(B \cdot W)$ are changing over time. The impulse responses are computed by fixing W_t at a desired time point. The impulse responses cab be generated for up to desired number of quarters and accumulate to assess how the Covid-19 impact is going to last under different scenarios.[2]

3. Empirical Methodology

Apart from value added growth rate (%) of the *n* major sectors two additional variables are used in the model: FORGDP, export-share weighted GDP growth rate of the trading partners of the economy studied and VISITOR, growth rate of visitor arrivals to the country. In addition, dummy variables to account for data outliers are also considered. Quarterly data upto 2019Q4 are used in the estimation of the pre-crisis parameter values.

Step 1

Estimating equation (3) requires the weights and then the weight matrix in (6) to account for interdependence among the sectors. One possibility is to use input-output tables from various years to work out the weights. Since weights should vary smoothly over time, a simple interpolation method can be used to fill the missing periods between input-output compilation years. Sometimes it is difficult to work out time varying weights. In such situations fixed weights may be used from a recent input-output table.

If input-output tables are not available fixed weights can be worked out directly from the sector value-added data as described below.

In the standard VAR framework, all the parameters are estimated from the observations of the *n* variables in the model. A two-step procedure can be used to obtain B and W in (6) separately from these estimates.

For illustration consider sector 1. The basic equation to estimate the weights is of the form:

$$y_{1t} = \phi_0 + \phi_1 y_{1t-1} + \phi_2 y_{1t-2} + \omega_2 y_{2t} + \omega_3 y_{3t} + \ldots + \omega_{10} y_{10t} + \lambda' Z_t + u_t \tag{9}$$

[2] Abeysinghe and Forbes (2005) discuss in detail the advantages of this type of SVAR model compared to the standard VAR framework.

where Z includes FORGDP, VISITOR and dummy variables to account for data outliers. Some experimentation is needed with these variables in the effort to obtain positive estimates for ω coefficients. If all the ω estimates are positive, then adjust them to sum to unity. But some ω values may turn out to be negative. Since weights cannot be negative, add the largest negative ω in absolute terms to all the ω coefficients and adjust them to sum to unity. This linear transformation does not change the relative position of the coefficients and the correlation between the original and transformed vectors is one. The adjusted ω's are the weights.

Step 2

After obtaining the weights, work out y_t^* in (3) and re-estimate the equation with two lags:

$$y_{1t} = \phi_0 + \phi_1 y_{1t-1} + \phi_2 y_{1t-2} + \beta_0 y_{1t}^* + \beta_1 y_{1t-1}^* + \beta_2 y_{1t-2}^* + \lambda' Z_t + u_t . \qquad (10)$$

Residual autocorrelation tests should indicate that two lags are sufficient. After estimating the equations for all the sectors B and W matrices for (6) can be compiled.

Step 3

The most difficult task in the exercise is calibrating the parameter values for the COVID-19 intervention dummy in (6) (Γ matrices). This requires generating forecasts for each sector upto the desired number of periods in order to calibrate the parameter values. Two exogenous variables in the model are FORGDP and VISITOR. Future values for these variables have to be assigned subjectively to generate the forecasts for the sectors. Forecast assumptions made on these two variables are explained in each chapter separately.

These two variables alone are not enough to generate forecast growth rates for the sectors. Accounting for sectoral interdependence is also important. Using the structure in (6) the forecasting model can be derived from:

$$(B_0 \cdot W) y_t = \phi_0 + (B_1 \cdot W) y_{t-1} + (B_2 \cdot W) y_{t-2} + \Lambda^* FORGDP + \Delta^* VISITOR_t + \varepsilon_t \qquad (11)$$

where Λ^* and Δ^* are diagonal matrices. Pre-multiplying (11) by $(B_0 \cdot W)^{-1}$ the forecasting model has the format:

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \Lambda FORGDP + \Delta VISITOR_t + u_t \qquad (12)$$

After forecasting sectoral growth rates for a desired number of quarters ahead and appending the data set with these values one more regression for each sector needs to be run. If the Covid-19 dummy is set to two lags, the regression is of the form:

$$y_{1t} = \phi_0 + \phi_1 y_{1t-1} + ... + \phi_2 y_{1t-p} + \gamma_0 D_t + \gamma_1 D_{t-1} + \gamma_2 D_{t-2} + v_t \; . \tag{13}$$

If parameter calibration starts from 2020Q1 then $D_t = 1$ for 2020Q1 and zero otherwise. Some back and forth experimentation with (12) and (13) is needed to calibrate subjectively reasonable γ values. The estimated γ values provide the calibrated parameter estimates for equation (6).

Step 4

After obtaining all the required numbers, use a dedicated software like SAS to generate the impulse responses as described in equation (8).

4. Conclusion

The forward-looking methodology presented in this chapter requires combining pre-crisis estimates with calibrated estimates for the intervention (Covid-19) effect to derive impulse responses from a Structural VAR model. For the calibration, forecasts of the interdependent variables (sectoral value-added growth rates) over the expected crisis period are obtained from the Structural VAR model with exogenous variables (FORGDP and VISITOR). Future values of the exogenous variables need to be set subjectively. After appending the data set with forecasts, an ARDL model similar to (2) is run to estimate the intervention parameters. Equipped with all the estimates (pre-crisis estimates and calibrated intervention estimates) the full VAR model is run to derived impulse responses from which direct and indirect growth effects can be generated as desired.

Apart from providing growth trajectories for each sector under the intervention effect, the analysis can also provide some projections to shed light on growth outlook under different scenarios. These are early warning lights to policy makers. The very objective of such warning lights is not to realize the projected bad outcome. Therefore, they should not be taken as forecasts.

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