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Central Bank Digital Currency with Adjustable Interest Rate in Small Open Economies*

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Abstract

We examine the economic consequences of an interest-bearing design of the Central-Bank Digital Currency (CBDC), and extend the discussion to an open-economy context with trade and capital flows. We use a dynamic stochastic general equilibrium (DSGE) model to simulate a baseline scenario with only a primary monetary policy rule, and two counter-factual scenarios with a primary monetary rule together with a secondary CBDC rule associated with adjustable interest-bearing CBDC — the price rule or the quantity rule. Our simulations show that 1) CBDC with an adjustable interest rate is welfare-improving; 2) a quantity rule delivers the best welfare outcome for society, but with uneven distributional effects between households and financial investors; 3) exchange rate movements and inflation are more stable with the adjustable interest rate; 4) imperfect substitutability between CBDC and bank deposits is the key for the effectiveness of using the CBDC as a secondary monetary policy instrument.

Keywords: CBDC, DSGE, monetary policy, blockchain, distributed ledger

JEL classification: E41, E42, E43, E52, E58, F31, F41

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1 Introduction

Digitisation of banking since the last decade has led to reduced usage of notes and coins for the purpose of transactions (see Figure 1). Whether central banks should ride the wave by digitising its money, commonly known as the central bank digital currency (CBDC), has become an increasingly popular topic of discussion.[1] In a working paper released by Bank of England, Barrdear and Kumhof (2016) define CBDC as:

> *... a universally accessible and interest-bearing central bank liability, implemented via distributed ledgers, that competes with bank deposits as medium of exchange...*

Of the features named in this definition, being a universally accessible central bank liability, and a competing medium of exchange with bank deposits, is nothing different from the narrow money, or cash, issued by the central bank. The distributed ledger technology constitutes the infrastructure facilitating payments in CBDC. Without diving into the technical details of the payment infrastructure, the interest-bearing feature is then the key economic innovation in central bank money. Kumhof and Noone (2018) highlight that the CBDC needs to bear an "adjustable interest rate", among other design principles, so as to serve as a secondary monetary tool. However, the extent to which an adjustable interest on central bank money will improve the policy outcomes remains unclear.

The present study aims to assess the macroeconomic implications of a CBDC with an adjustable interest rate, particularly in the context of an economy open to trade and capital flows. To this end, we propose a DSGE framework with CBDC and bank deposits, bearing differentiated interests, as competing media of exchange. In this framework, the central bank coordinates among fiscal, monetary and CBDC tools to achieve policy objectives. An exogenous foreign sector is introduced that interacts with the model economy's markets for consumption-goods and bonds so that uncertainties originating outside the home country are considered in designing monetary policies. This model allows us to easily toggle between a CBDC with a constant return, e.g. 0 per cent, and one with an adjustable interest rate following either a price rule that specifies the response of the nominal CBDC interest rate, or a quantity rule that specifies the response of the quantity of CBDC supply to macroeconomic conditions. Through simulations, we show that the added flexibility due to the adjustable CBDC interest is welfare-improving, especially in the presence of foreign shocks.

Our work contributes to the emerging literature of CBDC in four aspects. First, we are the first to formally examine CBDC in an open economy's context. The case of CBDC in an open economy is more complicated as compared to a standard closed economy such as Barrdear and Kumhof (2016). Issues such as granting access to foreign entities (Bank for International Settlements, 2018), and the transmission channel through the exchange rate (Meaning et al., 2018), all need to be carefully examined prior to actual implementation of CBDC. The design principles set out in Kumhof and Noone (2018) also disregard the exchange rate regimes. To our best knowledge,

[1] China is expected to launch CBDC in the near future as a response to Facebook's proposed digital currency Libra (Goh and Shen, 2019; Elegant, 2019). Small open economies like Sweden, Thailand, Turkey and Uruguay have already taken concrete steps or announced their intentions to launch a CBDC in the future (Desouza, 2019).

most existing discussions of CBDC with reference to open economy are qualitative. Our study therefore contributes to the literature by discussing CBDC in a framework with an exogenous foreign country.

Second, we assess the welfare implications of CBDC policy regimes. Keister and Sanches (2019) shows the optimal design policy of CBDC to be interest bearing using a new monetarist tradition model. In this paper, we are interested in the DSGE framework as it allows assessment of societal welfare due to alternative, counter-factual policies. Barrdear and Kumhof (2016) simulate the counter-factual scenarios using a DSGE model but do not provide a welfare assessment. In this paper, we simulate different combinations of conventional, and futuristic CBDC monetary policies. We provide both the welfare implications and the stabilising effects of these policy options.

Third, we assess in an unified framework the dynamic effects of the CBDC with adjustable interest and the constant or zero-interest CBDC on other assets such as bank deposits, bank loans and government securities. Interest-bearing CBDC has advantages in enhancing price stability (Bordo and Levin, 2017), and enabling negative interest rate (Rogoff, 2014). In addition, the CBDC also offers transaction efficiency compared with bank deposits. The attractiveness of the CBDC vis-á-vis bank deposits may have implications on the stability of the financial system and on the overall economy (Bank for International Settlements, 2018). Chiu et al. (2019) finds that interest-bearing CBDC can improve bank intermediation.

Fourth, we document that the independence of CBDC regimes is associated with its substitutability for bank deposits. Some existing studies, such as Barrdear and Kumhof (2016), assume that CBDC and bank deposits are imperfect substitutes, while others, such as Harrison and Thomas (2019), do not consider the existence of competing medium of exchange in the same framework. By conducting a sensitivity analysis on the substitutability between CBDC and bank deposits, we establish that CBDC with an adjustable interest rate may not be an effective monetary policy tool if it is a perfect substitute for bank deposits in providing transaction utility for the consumers.

The main findings of the paper are as follows. First, welfare of the society improves when the central bank follows an adjustable CBDC interest rate system. The welfare improvement is most profound in the presence of foreign shocks. Among the adjustable CBDC interest rate regimes considered, society experiences a larger welfare gain in the quantity rule regime when compared to the price rule regime. Second, we find distributional consequences associated with adjustable CBDC interest rates. The quantity rule regime provides a larger welfare gain for the households at the cost of welfare loss for financial investors. In contrast, financial investors witness a small welfare gain in the price rule regime. Third, exchange rate and inflation are found to be more stable in an environment of exogenous shocks when the CBDC interest rate is adjustable. Finally, imperfect substitutability between CBDC and deposits improves the central bank's ability in using CBDC as a secondary monetary policy instrument.

The remainder of the paper is organised as follows. Section 2 explains the transmission mechanism of CBDC monetary policies. Section 3 provides detailed descriptions of the model with CBDC. Section 4 discusses the results from our simulation exercises. Section 6 concludes.

2 Economics of CBDC monetary policies

The concept of an adjustable interest rate on central bank money has not been thoroughly explored, though it is not new. In his seminal book, Woodford (2003) describes a policy framework with a set of three monetary instruments in Section 3.2 of Chapter 2. This set of policy instruments includes the bond interest rate, the supply of monetary assets, and the interest rate of monetary assets. The money demand, after log-linearisation, is a function of the interest differential between bonds and monetary assets as the following:

$$\hat{m}_t = \eta_y \hat{Y}_t - \eta_i \left(\hat{i}_t - \hat{i}^m_t\right) + \epsilon_t \tag{1}$$

where $\hat{m}_t$ is real money balance, $\hat{Y}_t$ is output, $\hat{i}_t$ and $\hat{i}^m_t$ are interest rates of bonds and monetary assets, respectively, and η_y and η_i are elasticity parameters. It follows from the above equation that, to determine the money circulated in the economy, the central bank needs to specify both the interest rates on bonds and on monetary assets. Alternatively, the central bank may also specify the bond interest rate and the supply of monetary assets, letting the interest rate on monetary assets adjust to equate the demand and supply of money. In this scenario, the interest rate on monetary assets is equivalent to an adjustable interest rate on central bank money. However, because the central bank money held by the households have long been in physical form, the interest-bearing feature is costly to be implemented. Hence, the interest rate of monetary assets is often taken to be zero and unchanged.

Technology has advanced to the extent that central bank money may also exist in digital form like bank deposits. It is then feasible for a central bank to introduce an interest rate to households' holdings of central-bank issued digital money. Following the simple framework presented in Equation 1, and using the conventional Taylor rule as the mandatory monetary policy to specify $\hat{i}_t$, the central bank may adopt a secondary monetary policy which can be either a price-based rule that specifies the interest rate on monetary assets, or a quantity-based rule that specifies the supply of monetary assets. The last unspecified monetary policy instrument endogenously adjusts to clear the money market.

It is important to note that the money demand is associated differently with the two interest rates. Because the interest differential between bonds and monetary assets measures the opportunity cost in holding the latter, an increased demand for monetary assets is associated with a lower interest on bonds and/or a higher interest on monetary assets. As a result, to use either the price-based or the quantity-based measure as a secondary monetary policy tool to enhance the Taylor rule, the interest on monetary assets or the money supply entails a counter-cyclical component against the bond interest rate.

In Barrdear and Kumhof (2016), a price rule and a quantity rule are introduced. The price rule specifies the response of the interest rate on CBDC to inflation. It has two components. While one of it moves with the bond interest rate, the other component responds counter-cyclically to inflation: In times of high (low) inflation, the interest on CBDC decreases (increases) relative to bond interest rate, reducing (increasing) the attractiveness of holding money, hence withdrawing (injecting) liquidity from (into) the economy. Similarly, in a quantity rule, money supply is increased in times of lower inflation. The interest on monetary assets adjust upwards to clear the money market. We will analyse these two policies in Section 3.

In the context of an open economy, regimes with adjustable return on monetary assets are likely to lead to smaller movements in exchange rates. Because the interest rates on bonds and monetary assets move simultaneously to achieve policy objectives, it takes a smaller movement in the bond interest rate, as compared to a regime with fixed interest rate on monetary assets, to have the same effect on money supply. As a result, the interest rate differential between the domestic and foreign bonds is smaller, leading to a smoother movement in the nominal exchange rate via the uncovered interest parity.

In sum, CBDC serves as secondary tools to enhance the existing monetary policy framework. It could provide the central bank with greater flexibility in responding to macroeconomic conditions. Therefore, analysing the outcomes of these secondary tools delivers important policy implications for central banks in the digital era. In the next section, we describe a framework for analysing the CBDC monetary policies in a small open economy model.

3 Model

Our model features a small-open economy with deposits and CBDC being competing media of exchange. The domestic economy is a simplified variant of the Barrdear and Kumhof (2016) model. It retains the consumption loans in Barrdear and Kumhof (2016) as the means of credit creation in the economy. Without loss of generality, and for ease of identifying the transmission mechanism, we abstract from Barrdear and Kumhof (2016) the real-estate and capital-goods sectors. The domestic economy is small relative to the rest of the world in the sense that any disturbance in the domestic economy is negligible to the world, but not vice versa.

Figure 2 depicts the flows of funds among the sectors. As in Barrdear and Kumhof (2016), the model consists of households, financial investors, banks, firms, a central bank (government), and the world economy. Two types of agents live in an extended family, in which a fraction $1-\omega$ are the financially constrained households, and a fraction ω are the financially unconstrained financial investors. Both agents work in firms which produce consumption goods sold in both the domestic and global markets. They receive wages as reward for their labour.

The banks are relevant for credit creation. Bank deposits are created as households borrow from banks to finance their consumption needs. Bank deposits are also held by households and financial investors as part of their assets. In addition, households and financial investors hold assets in the form of CBDC and government bonds.

Among the three types of assets, deposits and CBDC provide liquidity and allow agents to use them for daily transactions. Bonds are relatively illiquid and are kept as a store of value.

The government issues CBDC in exchange for government bonds through open market operations. Government bonds are sold to (bought from) the public in exchange for CBDC when there is a need to reduce (increase) liquidity in the economy.

The interactions between the domestic and world economies are via two channels. Firstly, firms import intermediate capital goods as a production input, and sell final consumption goods to the world economy as exports. Secondly, financial investors borrow from the world economy by issuing foreign-currency denominated bonds (foreign debt) and pay the prevailing foreign interests.

In the remainder of this section, we describe the details of the model. All the variables given in the section below (in small letters) are in $real\ terms$ unless specified otherwise.

3.1 Banks

Banks follow the financing model of money creation where deposits are created as an output when banks make loans to households. Households use the loans to finance their consumption needs. The 'new money' through loans are credited to the households' deposit accounts. Banks are understood to face different exposure to non-credit risks, and are indexed by i.

The per-capita loan stocks held by households from bank i during period t to $t+1$ are given by $\ell_t^c(i)$ with the total loan stocks of bank i denoted by $\ell_t^\ell(i)$ where $\ell_t^\ell(i) = (1-\omega)\ell_t^c(i)$. The bank's real gross wholesale lending rates (including premium on cost of regulation) and real retail lending rates (including premium on credit risk) are given by r_t^ℓ and r_t^r, respectively. The bank can incur loan losses which are given by $\mathcal{L}_t^b(i) = (1-\omega)\mathcal{L}_t^c(i)$.

Both households and financial investors hold deposits with banks. The per capita deposit stocks of households and financial investors in bank i are denoted by $d_t^c(i)$ and $d_t^u(i)$, respectively. Total deposits in the bank i are given by $d_t(i) = (1-\omega)d_t^c(i) + \omega d_t^u(i)$. The bank's ex-post real deposit rate is denoted by r_t^d.

The balance sheet of bank i is

$$\ell_t^\ell(i) = d_t(i) + n_t^b(i) \tag{2}$$

where $n_t^b(i)$ refers to net worth of the bank. Net worth corresponds to the difference between gross return on loans ($r_t^\ell \ell_{t-1}^\ell(i)$) and expenses on interest payments on deposits ($r_{d,t}d_{t-1}(i)$), on loan losses ($\mathcal{L}_{t+1}^b(i)$), on ex-post cost of penalty ($\mathcal{M}_t^b(i)$) and dividends ($\delta^b n_t^b(i)$)[2]. Hence, net worth is expressed as:

$$n_t^b(i) = r_t^\ell \ell_{t-1}^\ell(i) - r_{d,t}d_{t-1}(i) - \mathcal{L}_t^b(i) - \delta^b n_t^b(i) - \mathcal{M}_t^b(i) \tag{3}$$

Bank i incurs the ex-post cost of penalty, $\mathcal{M}_t^b(i)$, only if the pre-dividend and pre-penalty net worth ($r_t^\ell \ell_{t-1}^\ell(i) - r_{d,t}d_{t-1}(i) - \mathcal{L}_t^b(i)$) falls below the minimum capital adequacy ratio (MCAR), which is a fraction (Υ) of the gross returns on the bank's loan book weighted for solvency risk. Should bank i fail to meet the MCAR, government regulations mandate that it pays a penalty as portion, χ, of its total loans ($\ell_t^\ell(i)$) at $t+1$.

The solvency risks of banks are incorporated into the model since the gross loan returns of banks are subject to an idiosyncratic shock $ln(\omega_{t+1}^b) \sim N\left(0, (\sigma^b)^2\right)$. The probability density function and cumulative density function of ω_{t+1}^b are given by $f_t^b(\omega_{t+1}^b) = f_{t+1}^b$ and $F_t^b(\omega_{t+1}^b) = F_{t+1}^b$, respectively. Since there is a continuum of i banks in the model, such a shock makes the banks heterogeneous in their loan book returns. This idiosyncratic risk can reflect a number of individual bank characteristics that are not directly related to standard banking operation of lending, such as differing success at raising non-interest income and minimizing non-interest expenses, where the sum of these over all banks equals zero.

The cut-off condition in which bank i would pay a penalty for falling below MCAR is given by

$$r_{t+1}^\ell \ell_t^\ell(i)\omega_{t+1}^b - r_{d,t+1}d_t(i) - \mathcal{L}_{t+1}^b(i) < \Upsilon\zeta^c r_{t+1}^\ell \ell_t^\ell(i)\omega_{t+1}^b \tag{4}$$

where ζ^c is the risk weight associated with consumer loans. The realized cutoff idiosyncratic shock ($\overline{\omega_t}^b$) below which the bank has to pay penalty is given by

$$\overline{\omega_t}^b = \frac{r_{d,t}d_{t-1}(i) + \mathcal{L}_t^b(i)}{(1-\Upsilon\zeta^c)r_t^\ell \ell_{t-1}^\ell(i)} \tag{5}$$

Hence, the (ex-post) penalty if paid by the bank is given by $\mathcal{M}_t^b(i) = \chi\ell_t^\ell(i)F_t^b(\overline{\omega}_{t+1}^b)$, noting that $E_t(\mathcal{L}_{t+1}^b(i)) = 0$.

The objective of the bank is to maximize its pre-dividend net worth given by

$$\max_{\ell_t^\ell(i), d_t(i)} E_t\left\{r_{t+1}^\ell \ell_t^\ell(i)\omega_{t+1}^b - r_{d,t+1}d_t(i) - \mathcal{L}_{t+1}^b(i) - \chi\ell_t^\ell(i)F_t^b\left(\overline{\omega}_{t+1}^b\right)\right\} \tag{6}$$

[2]In line with Barrdear and Kumhof (2016), we also assume that banks follow a fixed dividend policy where households and financial investors receive part of bankers' net worth at an exogenously fixed rate. This is based on the "extended family approach" of Gertler and Karadi (2011).

Substituting equation (2) to equation (6), and then taking the first order condition with respect to $\ell_t^\ell(i)$, we get

$$0 = E_t \left\{ r_{t+1}^\ell - r_{d,t+1} - \chi F_t^b \left(\overline{\omega}_{t+1}^b\right) - \chi f_t^b \left(\overline{\omega}_{t+1}^b\right) \frac{r_{d,t+1}}{r_{t+1}^\ell} \frac{\ell_t^\ell(i)}{n_t^b(i)} \frac{1}{(1-\Upsilon\zeta^c)\left[\frac{\ell_t^\ell(i)}{n_t^b(i)}\right]^2} \right\} \tag{7}$$

3.2 Households

The utility function of the household i depends on υ, which determines the degree of habit formation of lagged aggregate per capita consumption c_{t-1}^c in preferences. $c_t^c(i)$ is the per capita consumption of household i with θ_p the elasticity of substitution among j varieties of Dixit Stiglitz consumption aggregates given by $c_t^c(i) = \left[\int_0^1 c_t^c(i,j)^{\frac{\theta_p-1}{\theta_p}} dj\right]^{\frac{\theta_p}{\theta_p-1}}$. The utility also depends on hours worked $h_t^c(i)$ with η as labor supply elasticity. ψ_h refers to the weight of hours worked in the utility function. β_c is the discount factor.

The household maximizes its lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta_c^t \left\{ (1-\upsilon)\ln\left(c_t^c(i) - \upsilon c_{t-1}^c\right) - \psi_h \frac{h_t^c(i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \tag{8}$$

Subject to its budget constraint:

$$\begin{aligned} &c_t^c(i)(1+\tau_c)(1+s_t^c(i)) + (d_t^c(i) + m_t^c(i))\left(1+\phi_b(b_t^{rat} - \overline{b^{rat}})\right) - \Psi_{c,t}(i) \\ &- \ell_t^c(i)\left(1 - \frac{\varphi_c}{2}(\ell_t^c(i) - \ell_{t-1}^c(i))^2\right) = r_{d,t}d_{t-1}^c(i) + r_{m,t}m_{t-1}^c(i) \\ &\qquad + \Pi_t(i) + w_t^{hh}h_t^c(i)(1-\tau_L) + \frac{\iota}{1-\omega}\Omega_t(i) - \Gamma_t Col_{t-1}(i) \end{aligned} \tag{9}$$

The household pays taxes on consumption, τ_c, and incurs monetary transaction cost, $s_t^c(i)$, while spending on consumption, $c_t^c(i)$. It holds bank deposits, $d_t^c(i)$, which has a real return, $r_{d,t}$. The household also holds CBDC, $m_t^c(i)$, with real return, $r_{m,t}$, under the CBDC regime. In holding financial assets, the household incurs financial transaction costs, $\phi_b(b_t^{rat} - \overline{b^{rat}})$, where b_t^{rat} is government debt-to-GDP ratio and $\overline{b^{rat}}$ is its steady state value.[3] These financial transaction costs are treated as exogenous and rebated back to the household as lump-sum transfer, $\Psi_{c,t}(i)$. The household receives loans from the bank, $\ell_t^c(i)$, subject to quadratic adjustment costs, $(1 - \frac{\varphi_c}{2}(\ell_t^c(i) - \ell_{t-1}^c(i))^2)$, which impose a penalty on rapid changes in the level of loans.

Household income in the budget constraint comes from the following sources: returns from financial assets $(r_{d,t}d_{t-1}^c(i) + r_{m,t}m_{t-1}^c(i))$; profits from firms which the household owns $(\Pi_t(i))$; after tax wages from hours worked $(w_t^{hh}h_t^c(i)(1-\tau_L)$, where τ_L is the labour income tax, w_t^{hh} is real wage and $h_t^c(i)$ is hours of work); and a share (ι) of the real lump-sum income $(\Omega_t(i))$ accruing to the household. $\Gamma_t Col_{t-1}(i)$ in the budget constraint

[3] Barrdear and Kumhof (2016) note that empirical literature finds the level of government debt to be positively related to the equilibrium real interest rates. In addition, Schmitt-Grohé and Uribe (2007) among others, assume that transaction cost is a quadratic function of government debt-to-output ratio.

is the fraction (Γ_t) of the collateral value ($Col_{t-1}(i)$) paid by household to the bank to cover the loan interest payments.

Monetary transaction costs of household i, $s_t^c(i)$, follow the specification of Schmitt-Grohé and Uribe (2007) expressed as

$$s_t^c(i) = A_c \upsilon_t^c(i) + \frac{B_c}{\upsilon_t^c(i)} - 2\sqrt{A_c B_c} \tag{10}$$

$\upsilon_t^c(i)$ is velocity and given by

$$\upsilon_t^c(i) = \frac{c_t^c(i)(1+\tau_c)}{f_t^c(i)} \tag{11}$$

$f_t^c(i)$ refers to liquidity generating function of the household and given by

$$f_t^c(i) = (d_t^c(i))^\theta + (T^{fintech} m_t^c(i))^\theta \tag{12}$$

where $\frac{1}{1-\theta}$ is the elasticity of substitution between CBDC and deposits. $T^{fintech}$ is a relative productivity coefficient for CBDC which makes monetary transactions more efficient while using CBDC. Hence, the household will hold CBDC even if the CBDC interest rate is lower than the bank deposit interest rate.

Collateral pledged by the household to secure loans, ($Col_t(i)$), includes returns on financial assets such as deposits ($r_{d,t+1} d_t^c(i)$) and CBDC ($r_{m,t+1} m_t^c(i)$) in addition to real asset returns of annualized current labour income ($r_{n,t+1} w_t^{hh} h_t^c(i)(1-\tau_L)$). $r_{n,t}$ is the return on nominal cash flow and equal to $\frac{1}{\pi_t}$. Hence,

$$Col_t(i) = \kappa_t^r \kappa_t^f (r_{d,t+1} d_t^c(i) + r_{m,t+1} m_t^c(i)) + \kappa_t^r r_{n,t+1} w_t^{hh} h_t^c(i)(1-\tau_L) \tag{13}$$

where κ_t^f is the coefficient for the financial collateral and κ_t^r is coefficient on the bank's willingness to lend based on the collateral.

The collateral value given in Equation 13 is subject to an idiosyncratic shock which changes $Col_t(i)$ to $\omega_{t+1}^c Col_t(i)$ in beginning of period $t+1$, where $ln(\omega_{t+1}^c) \sim N(0,(\sigma^c)^2)$. The density function and cumulative density function of ω_{t+1}^c are $f_{t+1}^c = f_t^c(\overline{\omega}_{t+1}^c)$ and $F_{t+1}^c = F_t^c(\overline{\omega}_{t+1}^c)$, respectively.

According to a standard debt contract, the household receives the loan amount $\ell_t^c(i)$ at a gross nominal retail interest of i_t^r. The nominal retail interest rate i_t^r is pre-committed at t and not determined at $t+1$. This makes expected profits zero for banks but the realized ex post profits are different from zero.

Borrowers who experience ω_{t+1}^c below a cut off level $\overline{\omega^c}_{t+1}$ are unable to pay the committed retail interest rate and enter into bankruptcy. All the assets of such borrowers must be handed over to the bank. However, the bank incurs monitoring costs on their lending activity. Hence, the bank is able to recover only a fraction $(1-\xi)$ of the collateral value. The bank's ex ante zero profit condition is given by

$$E_t \left\{ \left[(1 - F_t(\overline{\omega}^c_{t+1})) r^r_{t+1} \ell^c_t(i) + (1 - \xi) \int_0^{\overline{\omega}^c_{t+1}} Col_t(i) \omega^c f^c_t(\omega^c) d\omega^c \right] - r^\ell_{t+1} \ell^c_t(i) \right\} = 0 \tag{14}$$

The terms in square brackets refer to the total payoff from lending: the gross retail interest payments of borrowers on their loans if the borrowers' $\omega^c_{t+1} \geq \overline{\omega^c}_{t+1}$; and the recovery from collateral assets if $\omega^c_{t+1} < \overline{\omega^c}_{t+1}$. $\overline{\omega}^c_t$ is determined by equating the gross interest payments if borrower continues to operate to the gross returns on collateral assets if the borrower becomes bankrupt

$$\overline{\omega}^c_t = \frac{r^r_t \ell^c_{t-1}(i)}{Col_{t-1}(i)} \tag{15}$$

We can rewrite Equation 14 as

$$0 = E_t \{ Col_t(i) \left[\Gamma_{t+1} - \xi G_{t+1} \right] - r^\ell_{t+1} \ell^c_t(i) \} \tag{16}$$

where by definition, the bank's gross share in the value of collateral is:
$\Gamma_{t+1} = \Gamma_t(\overline{\omega}^c_{t+1}) = \overline{\omega}^c_{t+1} \int_{\overline{\omega}^c_{t+1}}^{\infty} f_t(\omega^c_{t+1}) d\omega^c_{t+1} + \int_0^{\overline{\omega}^c_{t+1}} \omega^c_{t+1} f^c_t(\omega^c_{t+1}) d\omega^c_{t+1}$; and
the proportion of collateral value that the lender has to spend on monitoring costs is:
$\xi G_{t+1} = \xi G_t(\overline{\omega}^c_{t+1}) = \xi \int_0^{\overline{\omega}^c_{t+1}} \omega^c_{t+1} f^c_t(\omega^c_{t+1}) d\omega^c_{t+1}$.

Real lump-sum income (Ω_t), a portion (ι) of which accrue to households, consists of dividends from banks ($\delta^b n^b_t$), transfers from the government (trf_t), lump sum tax paid to the government (τ^{ls}_t), the lump-sum component (ς) of the monitoring costs ($\mathcal{M}_t$) and the aggregate monetary transactions costs ($\mathcal{T}_t$).

$$\Omega_t = \delta^b n^b_t + trf_t - \tau^{ls}_t + (1 - \varsigma)(\mathcal{M}_t + \mathcal{T}_t) \tag{17}$$

where $\mathcal{T}_t = (1 - \omega) c^c_t (1 + \tau_c) s^c_t$

The representative household maximizes Equation 8 subject to Equation 9 and Equation 16. We get the first order conditions after taking into account symmetry across households as follows

$$c^c_t : \frac{(1 - \upsilon)}{c^c_t - \upsilon c^c_{t-1}} = \lambda_{1,t} (1 + \tau_c)(1 + s^c_t + s^{cc'}_t \upsilon^c_t) \tag{18}$$

where $s^{cc'}_t = A_c \frac{\upsilon^c_t}{c^c_t} - \frac{B_c}{c^c_t \upsilon^c_t}$

$$\ell^c_t : \lambda_{1,t} \left(1 - \frac{\varphi_c}{2} (\ell^c_t - \ell^c_{t-1})^2 - \varphi_c \ell^c_t (\ell^c_t - \ell^c_{t-1}) \right) = \beta_c E_t \left\{ \lambda_{1,t+1} r^\ell_{t+1} \lambda_{2,t+1} \right\} \tag{19}$$

where $\lambda_{2,t+1} = \frac{\Gamma'_{t+1}}{(\Gamma'_{t+1} - \xi G'_{t+1})}$ and Γ'_{t+1} and G'_{t+1} are derivatives of Γ_{t+1} and G_{t+1} with respect to $\overline{\omega}^c_{t+1}$.

$$h^c_t : \psi_h (h^c_t)^{\frac{1}{\eta}} = \lambda_{1,t} w^{hh}_t (1-\tau_L) \\ + \beta_c E_t \left\{ \lambda_{1,t+1} r_{n,t+1} w^{hh}_t (1-\tau_L) \left(-\kappa^r \Gamma_{t+1} + \lambda_{2,t+1} \kappa^r (\Gamma_{t+1} - \xi G_{t+1}) \right) \right\} \quad (20)$$

$$d^c_t : \lambda_{1,t} \left(1 + \phi_b (b^{rat}_t - \overline{b^{rat}}) + c^c_t (1+\tau_c) s^{cd'}_t \right) = \\ \beta_c E_t \left\{ \lambda_{1,t+1} r_{d,t+1} \left[\left(1 - \kappa^r \kappa^f \Gamma_{t+1} \right) + \lambda_{2,t+1} \kappa^r \kappa^f (\Gamma_{t+1} - \xi G_{t+1}) \right] \right\} \quad (21)$$

where $s^{cd'}_t = -A_c \frac{\upsilon^c_t}{f^c_t} f^{cd'}_t + \frac{B_c}{\upsilon^c_t f^c_t} f^{cd'}_t$, $f^{cd'}_t = \theta (d^c_t)^{\theta-1}$

$$m^c_t : \lambda_{1,t} \left(1 + \phi_b (b^{rat}_t - \overline{b^{rat}}) + c^c_t (1+\tau_c) s^{cm'}_t \right) = \\ \beta_c E_t \left\{ \lambda_{1,t+1} r_{m,t+1} \left[\left(1 - \kappa^r \kappa^f \Gamma_{t+1} \right) + \lambda_{2,t+1} \kappa^r \kappa^f (\Gamma_{t+1} - \xi G_{t+1}) \right] \right\} \quad (22)$$

where $s^{cm'}_t = -A_c \frac{\upsilon^c_t}{f^c_t} f^{cm'}_t + \frac{B_c}{\upsilon^c_t f^c_t} f^{cm'}_t$, $f^{cm'}_t = \theta \left(T^{fintech} m^c_t \right)^{\theta-1}$

The monetary frictions feature prominently in the above optimal conditions. Equation 18 shows that household expenditures on consumption goods are higher by a ”mark up” of $(1 + s^c_t + s^{cc'}_t \upsilon^c_t)$ with monetary frictions than without it. Barrdear and Kumhof liken this markup to a ”liquidity” tax with a distortionary tax rate, $\tau^{liq}_{c,t}$, expressed as

$$\tau^{liq}_{c,t} = 1 + s^c_t + s^{cc'}_t \upsilon^c_t \quad (23)$$

They note the likeness between monetary frictions and distortionary taxes on their impact on real economy as shown in the optimal conditions. Furthermore, Equation 18 shows the distortions caused by changes in $\tau^{liq}_{c,t}$. Similarly, if Equation 18 is combined with Equation 20, the resulting equation expressing the marginal rate of substitution of consumption and labor would show the distortions caused by changes in $\tau^{liq}_{c,t}$. Hence, due to the monetary frictions, changes in the quantity of bank deposits and in the quantity of CBDC or in the quantity of monetary transaction balances get transmitted to the real economy.

3.2.1 Foreign sector

As in Galí and Monacelli (2005), the economy with the global city is small, and foreigners are assumed to live in a large economy. Trade flows depend on exports of domestic goods to foreigners (exp_t) and by imports of intermediate goods from foreigners (k^*_t). Capital flows include borrowing by financial investors from the foreign sector in the form of financial investors issuing foreign-currency denominated bonds (b^*_t). Import price $f^{k^*}_t$, export demand exp_t and foreign interest rate i_t^* are exogenously determined in the model.

3.3 Firms

The production sector in the economy is owned by the households. The labor to the firm is supplied by both households and financial investors. The representative firm sells the output to households, financial investors and the foreign sector (exports). Since there is a continuum of output variety populating the unit interval, we can index the variety by i. The final good (y_t) is a CES aggregate of these output varieties.

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\theta_p - 1}{\theta_p}} di \right)^{\frac{\theta_p}{\theta_p - 1}} \tag{24}$$

The demand function of each output variety is given by

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta_p} y_t \tag{25}$$

The Cobb Douglas production function for the output variety i is given below.

$$y_t(i) = A_t (k_t^*(i))^{1-\alpha} h_t(i)^{\alpha} \tag{26}$$

where $h_t(i)$ is the labor input and $k_t^*(i)$ is the imported intermediate good input for production of good i. A_t is the factor productivity of the firm. The firm pays wages w_t^{hh} for the labor input and q_t for the imported intermediate good. The intermediate good input is imported at foreign import price $f_t^{k^*}$ which is exogenously determined in our model. Intermediate good price in domestic currency, q_t, is obtained through the below equation.

$$q_t = \frac{e_t f_t^{k^*}}{P_t} \tag{27}$$

where e_t is the nominal exchange rate and P_t is the domestic price. q_t is also defined as real exchange rate in our model.

We follow the Calvo (1983) pricing in our model where there is a probability $1 - \phi$ that the firm resets its price in a given period. Thus with probability ϕ, firms keep their prices unchanged. Let $\tilde{P}_t(i)$ denote the price set by a firm i adjusting its price in period t. Using the Calvo price setting model, $P_{t+s}(i) = \tilde{P}_t(i)$ with probability ϕ^s for $s = 0, 1, 2....$. We drop the i subscript as all firms optimising the price in any given period will choose the same price. In line with Galí and Monacelli (2016), the objective of the firm while setting the price in period t is to maximize the present value of its profits.

$$Max:\sum_{s=0}^{\infty}\phi^{s}\,E_t\left\{M_{t,t+s}\left[\tilde{P}_tY_{t+s|t}-\mathcal{C}(Y_{t+s|t})\right]\right\} \tag{28}$$

subject to the below demand constraint

$$Y_{t+s|t}=\left(\frac{\tilde{P}_t}{P_{t+s}}\right)^{-\theta_p}Y_{t+s} \tag{29}$$

where $M_{t,t+s}\equiv\beta_c^s\frac{\lambda_{1,t+s}}{\lambda_{1,t}}$ (see Equation 18) is the stochastic discount factor, $Y_{t+s|t}$ is the nominal output produced by a firm in period $t+s$ who last reset its price in period t and $\mathcal{C}(.)$ is the nominal cost function. We get the below price setting equation of the firm after optimisation.

$$\sum_{s=0}^{\infty}\phi^{s}E_t\left\{M_{t,t+s}Y_{t+s|t}\left(\tilde{P}_t-\frac{\theta_p}{\theta_p-1}\mathcal{MC}_{t+s|t}\right)\right\} \tag{30}$$

where $\mathcal{MC}_{t+s|t}$ is the nominal marginal cost of the firm which last reset its price in period t.

We use Alba et al. (2019) to rewrite the above price setting equation recursively using the below three equations.

$$F_{1,t}=Y_t+\phi M_{t,t+1}(\pi_{t+1})^{\theta_p}F_{1,t+1} \tag{31}$$

$$F_{2,t}=Y_t\mathcal{MC}_t^r+\phi M_{t,t+1}(\pi_{t+1})^{1+\theta_p}F_{2,t+1} \tag{32}$$

$$\left(\frac{\tilde{P}_t}{P_t}\right)F_{1,t}=\frac{\theta_p}{\theta_p-1}F_{2,t} \tag{33}$$

$\mathcal{MC}_t^r\equiv\left(\frac{q_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{w_t^{hh}}{\alpha}\right)^{\alpha}\frac{1}{A_t}$ denotes the real marginal cost of a firm. $\pi_{t+1}\equiv\frac{P_{t+1}}{P_t}$ is the domestic price inflation between t and t+1. We define $F_{1,t}\equiv E_t\left\{\sum_{s=0}^{\infty}\phi^sM_{t,t+s}Y_{t+s}\left(\prod_{k=1}^{s}\pi_{t+k}\right)^{\theta_p}\right\}$ and $F_{2,t}\equiv E_t\left\{\sum_{s=0}^{\infty}\phi^sM_{t,t+s}Y_{t+s}MC_t^r\,(\prod$
We also have,

$$P_t^{1-\theta_p}=\phi P_{t-1}^{1-\theta_p}+(1-\phi)\tilde{P}_t^{1-\theta_p}\implies 1=\phi\pi_t^{\theta_p-1}+(1-\phi)\left(\frac{\tilde{P}_t}{P_t}\right)^{1-\theta_p} \tag{34}$$

3.4 Financial Investors

Like the households, the utility function of the financial investors also depends on υ which determines the degree of habit formation of lagged average per capita consumption c_{t-1}^u in preferences. Utility also depends on hours worked h_t^u with η as labour supply elasticity. ψ_h refers to the weight of hours worked in the utility function. β_c is the discount factor. The utility function of financial investor differs from the household with respect to the presence of monetary transactions balances (f_t^u) in financial investors' utility function with weight ψ_f.The interest semi-elasticity of deposit demand by financial investors is determined by ϑ. Interest semi-elasticity of deposit demand is defined as the percentage change in demand for bank deposits resulting from a one percentage point increase in the opportunity cost of bank deposits.

The objective of the financial investor is to maximize the utility function

$$Max : E_0 \Sigma_{t=0}^{\infty} \beta_u^t \left\{ (1-\upsilon) log(c_t^u(i) - \upsilon c_t^u) - \psi_h \frac{h_t^u(i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi_f \frac{(f_t^u(i))^{1-\frac{1}{\vartheta}}}{1-\frac{1}{\vartheta}} \right\} \tag{35}$$

where $c_t^u(i) = \left[\int_0^1 c_t^u(i,j)^{\frac{\theta_p - 1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p - 1}}$ and $f_t^u = d_t^{u\theta}$.

Domestic government bonds b_t^u and bank deposits d_t^u are the real assets owned by financial investors. Similar to the framework in Schmitt-Grohé and Uribe (2003), our model has foreign debt in the form of foreign-currency denominated bonds issued by the financial investors in international markets ($b_t^{u*}(i)$). r_t^* corresponds to the real foreign interest rate defined by $r_t^* = \frac{i_{t-1}^*}{\pi_t}$, where i_t^* is the nominal interest rate at which financial investors can borrow in international markets in period t. The consumption expenses of the financial investors amount to $c_t^u(i)(1+\tau_c)$ and their after tax income equals $w_t^{hh} h_t^u(i)(1-\tau_L)$. In addition to wage income, they also receive a share $(1-\iota)$ of the lump-sum income Ω_t. The budget constraint faced by the financial investors is given below.

$$\begin{aligned}(b_t^u(i) + d_t^u(i)) \left(1 + \phi_b(b_t^{rat} - \overline{b^{rat}})\right) + q_t r_t^* \zeta_t^e b_{t-1}^{u*}(i) = q_t b_t^{u*}(i) + \Psi_{u,t}(i) + r_t b_{t-1}^u(i) + r_{d,t} d_{t-1}^u(i) \\ - c_t^u(i)(1+\tau_c) + w_t^{hh} h_t^u(i)(1-\tau_L) + \frac{1-\iota}{\omega}\Omega_t \end{aligned} \tag{36}$$

ζ_t^e is the risk premium following Schmitt-Grohé and Uribe (2007). It is a positive convex function of foreign debt to aggregate output ratio to prevent unlimited borrowing from abroad.

$$\zeta_t^e = exp\left[\varphi_e \left(\frac{q_t b_t^{u*}}{\overline{gdp}} - \frac{\overline{q}\overline{b^{u*}}}{\overline{gdp}} \right) \right] \tag{37}$$

The transaction costs associated with holding domestic assets such as government bonds and deposits, $\phi_b(b_t^{rat} - \overline{b^{rat}})(b_t^u(i) + d_t^u(i))$, is rebated back to financial investors as a lump sum transfer $\Psi_{u,t}(i)$. Financial investors maximize Equation 35 subject to Equation 36. Since all financial investors face the same initial endowment, we can drop i index and we get the below first order conditions.

$$c_t^u : \frac{(1-\upsilon)}{c_t^u - \upsilon c_{t-1}^u} = \lambda_{3,t}(1-\tau_c) \tag{38}$$

$$h_t^u : \psi_h (h_t^u)^{\frac{1}{\eta}} = \lambda_{3,t} w_t^{hh}(1-\tau_L) \tag{39}$$

$$b_t^u : \beta_u \lambda_{3,t+1} r_{t+1} = \lambda_{3,t}\left(1 + \phi_b(b_t^{rat} - \overline{b^{rat}})\right) \tag{40}$$

$$b_t^{u*} : \beta_u \lambda_{3,t+1} q_{t+1} \zeta_{t+1}^q r_{t+1}^* = \lambda_{3,t} q_t \tag{41}$$

$$d_t^u : \beta_u \lambda_{3,t+1} r_{d,t+1} = \lambda_{3,t}\left(1 + \phi_b(b_t^{rat} - \overline{b^{rat}})\right) - \psi_f \theta (d_t^u)^{\theta - 1 - \frac{\theta}{\vartheta}} \tag{42}$$

3.5 Fiscal policy

The government budget constraint is given by

$$b_t + m_t = r_t b_{t-1} + r_{m,t} m_{t-1} + g_t + trf_t - \tau_t^{ls} - \tau_L w_t^{hh} h_t - \tau_c c_t \tag{43}$$

where b_t and m_t are government supply of bonds and CBDC, respectively, and government spending g_t is a fraction s_g of gdp. Hence, $g_t = s_g gdp_t$.

gdp_t is defined as

$$gdp_t = c_t + g_t + exp_t \tag{44}$$

Following the fiscal policy rule of Barrdear and Kumhof (2016), we also protect the government budget from potentially high volatile seigniorage revenue through CBDC creation.

$$gdx_t^{rat} = \overline{gdx^{rat}} - \varpi_{b,gdp} ln\left(\frac{gdp_t}{\overline{gdp}}\right) \tag{45}$$

where gdx_t^{rat} is the adjusted budget deficit ratio and given by $gdx_t^{rat} = \frac{(b_t - b_{t-1}) + (m_t - m_{t-1})}{gdp_t}$. $\varpi_{b,gdp}$ is the response coefficient which allows the deficit to fluctuate when there is an output gap, $ln\left(\frac{gdp_t}{\overline{gdp}}\right)$. Equation 45 is assumed to endogenise lump sum tax, τ_t^{ls}.

3.6 Monetary policy

The monetary policy follows the conventional (nominal) interest rate rule of Barrdear and Kumhof (2016) which includes interest rate smoothing and deviations of current period inflation from target inflation, but modified to include deviations of real exchange rate and output from their steady state values.

$$i_t = (i_{t-1})^{\rho_i} \left(\frac{1 + \phi_b(brat_t - \overline{brat})}{\beta_u} \right)^{1-\rho_i} \left(\frac{\pi_t}{\overline{\pi}} \right)^{(1-\rho_i)\varpi_{i,\pi}} \left(\frac{q_t}{\overline{q}} \right)^{(1-\rho_i)\varpi_{i,q}} \left(\frac{gdp_t}{\overline{gdp}} \right)^{(1-\rho_i)\varpi_{i,gdp}} \quad (46)$$

where i_t is nominal interest rate, ϕ_b determines the elasticity of real-interest-rate to government debt-to-GDP ratio ($brat_t$), ρ_i is the smoothing parameter associated with nominal interest rate and $\varpi_{i,\pi}$, $\varpi_{i,q}$ and $\varpi_{i,gdp}$ are the feedback coefficients on inflation, real exchange rate and output, respectively. $\overline{brat}$, $\overline{\pi}$, $\overline{q}$ and $\overline{gdp}$ refer to the steady state values of government debt-to-GDP ratio, inflation, real exchange rate and gdp, respectively.

CBDC regimes Given the interest rate rule in Equation 46, the CBDC may be used as a secondary monetary policy instrument to maintain price stability. The secondary policy instrument involves using either a CBDC price rule or a CBDC quantity rule as described below.

1. **Baseline regime**

   The baseline CBDC regime assumes the central bank only follows the conventional interest rate rule in Equation 46 so the nominal CBDC interest rate is fixed at its steady level, $\overline{i_m}$:

$$i_{m,t} = \overline{i_m} \quad (47)$$

2. **Price Rule Regime**

   The price rule regime assumes the central bank uses a secondary monetary policy instrument based on the following rule:

$$i_{m,t} = (i_{m,t-1})^{\rho_{i^m}} \left[\frac{i_t}{\mathbf{s_p}} \left(\frac{\pi_t}{\overline{\pi}} \right)^{(-\varpi_{i^m,\pi})} \right]^{(1-\rho_{i^m})} \quad (48)$$

   where ρ_{i^m} is the smoothing parameter of price rule, $\mathbf{s_p}$ is the steady state spread between risk free bond rate and CBDC interest rate and $\varpi_{i^m,\pi}$ is the price rule inflation feedback coefficient. Equation 48 indicates that when inflation increases above the target, ($\pi_t > \overline{\pi}$), the CBDC interest rate, $i_{m,t}$, declines relative to the interest rate, i_t, making CBDC less attractive relative to government bonds. Households demand less CBDC so the central bank reduces CBDC held by households and supplies more government bonds to financial investors. As CBDC is exchanged for government bonds, liquidity is endogenously reduced. This reduction in liquidity would have effects beyond that of the policy interest rate, i_t, specified in Equation 46, since households could only replace their reduced CBDC holdings with additional bank deposits. However, bank deposits are more expensive and are imperfect substitutes of CBDC so financial transaction costs,

$\phi_b(b_t^{rat} - \overline{b^{rat}})$, and "liquidity" taxes, $\tau_{c,t}^{liq}$, increase causing real activity to decrease.

3. **Quantity Rule Regime**

   Under the quantity rule regime, the central bank adjusts the CBDC-to-GDP ratio (m_t^{rat}) according to

$$m_t^{rat} = \overline{m^{rat}} - \varpi_{m,\pi} \ln\left(\frac{\pi_t}{\overline{\pi}}\right) \tag{49}$$

   where $m_t^{rat} = \frac{m}{gdp}$ and $\varpi_{m,\pi}$ is the quantity rule inflation feedback coefficient. The rule states that when inflation is expected to be above target ($\pi_t > \overline{\pi}$), the central bank reduces CBDC from the economy through open market sales of government bonds. This reduces the amount of liquidity in the economy. The impact of reduced liquidity on the real economy is the same as in the price-rule regime. Since the supply of CBDC decreases relative to demand of CBDC, and CBDC is an imperfect substitutes to other assets, the remaining CBDC circulating in the economy becomes more valuable. Therefore, households would be willing to hold CBDC with lower returns so the CBDC interest rate, $i_{m,t}$, decreases.

3.7 Effectiveness of CBDC monetary policy

Before closing the model, we highlight that the effectiveness of the central bank's CBDC monetary policy may depend on its ability to autonomously set the CBDC interest rate from the deposit rate. A central bank would only have some autonomy on the CBDC interest rate when the CBDC is not a perfect substitute for bank deposits. By perfect substitute, we mean that both currencies deliver the same utility to households. Barrdear and Kumhof (2016) assume that CBDC and bank deposits are not perfect substitutes because CBDC features better transaction facilities than bank deposits. Such assumption may or may not hold in reality. The central bank therefore needs to be aware of the consequence should consumers be indifferent between the two media of exchange.

The log linear approximations of Equation 21 and Equation 22 yield the following relationship between the rates on deposits and CBDC.[4]

$$\hat{i}_{d,t} - \hat{i}_{m,t} = (1-\theta)\left(\hat{m}_t^c - \hat{d}_t^c\right) \tag{50}$$

where $i_{m,t}$ and $i_{d,t}$ refer to nominal CBDC rate and nominal deposit rate, respectively. The elasticity of substitution between CBDC and deposits equal $\frac{1}{1-\theta}$ via Equation 12.

In the case of CBDC and deposits being perfect substitutes, then $\theta = 1$.

$$\hat{i}_{m,t} = \hat{i}_{d,t} \tag{51}$$

[4] We use $\phi_b \to 0$ to derive the equation.

It is evident from Equation 51 that the nominal CBDC interest rate would simply reflect the changes in nominal deposit rate when CBDC and deposits are perfect substitutes. This would constrain the central bank from using nominal CBDC rate or the CBDC-to-GDP ratio as secondary monetary policy instruments. Hence, CBDC and deposits must be imperfect substitutes ($\theta < 1$) so that the central bank could use CBDC as secondary monetary policy instrument under the price rule and quantity rule regimes. We explore this in section 5 to explain the role of θ in the performance of CBDC monetary policy.

3.8 Market Clearing

The equilibrium conditions of our model are detailed in this sub-section. In the domestic consumption goods market, the output of goods produced by the firms for domestic consumption equal the demand from both households and financial investors.

$$c_t = \omega c_t^u + (1 - \omega)c_t^c \tag{52}$$

The labour supply from households and financial investors are assumed to be homogeneous. The market clears when labour demand from firms equal labour supply from both households and financial investors.

$$h_t = \omega h_t^u + (1 - \omega)h_t^c \tag{53}$$

The market for domestic government bonds and CBDC clear when the demand for bonds from financial investors and the demand for CBDC from households equal the government's supply of bonds and CBDC respectively.

$$b_t = \omega b_t^u \tag{54}$$

$$m_t = (1 - \omega)m_t^c \tag{55}$$

In the banking sector, we have equilibrium conditions associated with deposits and loans. In equilibrium, the supply of deposits from banks equal the demand for deposits from both households and financial investors. On the other hand, loan market clears when the banks' loan supply covers the households' demand for credit.

$$d_t = \omega d_t^u + (1 - \omega)d_t^c \tag{56}$$

$$\ell_t = (1 - \omega)\ell_t^c \tag{57}$$

The goods market is in equilibrium when the total output of firms equal the demand for consumption goods from both households and foreigners, government spending on goods and real resource cost on monitoring and monetary transactions (see Equation 17).

$$(1-\omega)y_t = c_t + g_t + exp_t + \varsigma(\mathcal{M}_t + \mathcal{T}_t) \tag{58}$$

Finally, we have the below balance of payments identity where the trade balance measures the change in the net foreign liabilities position of the economy.

$$q_t\omega b_t^{u*} = q_t\zeta_t^e r_t^*\omega b_{t-1}^{u*} + q_t(1-\omega)k_t^* - exp_t \tag{59}$$

3.9 Shocks

The exogenous variables, $\mathbf{e}_t = \{A_t, exp_t, i_t^*\}$ are assumed to follow an autoregressive process of order 1.

$$\log\mathbf{e}_t = \rho_{\mathbf{e}}\log(\mathbf{e}_t) + (1-\rho_{\mathbf{e}})\log(\overline{\mathbf{e}}) + \xi_{\mathbf{e},t}; \qquad \xi_{\mathbf{e},t} \sim N(0,\sigma_{\mathbf{e}}^2) \tag{60}$$

3.10 Calibration

We solve the the model numerically, and then generate the impulse responses and calculate the welfare function using Dynare. It is not our intention to calibrate the model to a specific country. We are instead keen on calibrating the parameters based on a prototype small open economy. The time period is a quarter.

3.10.1 Model Parameterisation

We summarise in Table 1 the parameters, the parameter values and the sources of the parameter values. We take the values of A_c and B_c from Bacchetta and Perazzi (2018) who estimate Schmitt-Grohé and Uribe (2004) monetary transaction cost parameters for a small open economy. The value of the labour share α is standard in the literature. We follow Galí and Monacelli (2005) to fix the values of η and ϕ. In line with Galí and Monacelli (2005), θ_p is calibrated to imply a steady state price markup of 1.2. We follow Benes et al. (2015) to set the annual nominal policy interest rate, i_t, as 1% in the steady state by assuming $\beta_u = 0.9975$. We calibrate β_c to imply a steady state spread of 0.17% between nominal policy interest rate, i_t, and nominal deposit interest rate, i_t^d.[5] This is close to the average deposit and policy interest rate spread in New Zealand, which we consider as a prototype small open economy.

We take the values of ψ_f and ψ_h from Lozej et al. (2018) who estimate the utility weight of deposits for the small open economy of Ireland. We set the value of ϕ_b to be consistent with the estimate of short term nominal policy interest rate elasticity with respect to government debt to GDP ratio in Sweden by Lindé (2001). The values

[5] Barrdear and Kumhof (2016) conduct a similar exercise to calibrate β_c for the United States. We use quarterly data on deposit and central bank policy rate of New Zealand from *International Financial Statistics*. The average spread between policy interest rate and deposit rate from 2000:Q1 to 2007:Q4 in New Zealand is 0.17%.

of ϑ, s_g and υ are obtained from Jacob and Munro (2018) who estimate these parameters for New Zealand. We use the value of elasticity of deposit demand estimated by Jacob and Munro (2018) as the elasticity of financial investor deposit demand parameter, ϑ. We follow Benes et al. (2015) to fix the risk premium coefficient, φ_e and consumption loan adjustment cost coefficient, φ_c. Consumption tax, τ_c, of 15% corresponds to the flat rate of goods and service tax in New Zealand.[6] We set the labour income tax, τ_L, from our observation that majority of New Zealand population is taxed at 30% of personal income.[7]

The value of parameters ω, ι, ς, κ_f, θ, T^{fintec} and ξ are consistent with the estimates of Barrdear and Kumhof (2016). The bank riskiness parameter, σ_b, is estimated based on the assumption that 2%[8] of banks violate Minimum Capital Adequacy Ratio (MCAR) at the steady state. MCAR parameter Υ at 8% is fixed as per the Basel II and Basel III regulations. In accordance with the Basel III mandate for capital conservation buffer at 2.5%, we assume that the banks' actual capital adequacy ratio equal 10.5%[9] in the steady state to calibrate the value of δ_b. We also set the risk weight of consumption loan, ζ^c, at 0.75 by following the Basel-III regulations.

The rest of the banking sector parameters namely χ, σ_c and κ^r are calibrated to match the observed properties of the New Zealand Banking system. We calibrate χ to imply a steady state spread of 1.5% between the nominal loan interest rate, i_t^ℓ, and the nominal deposit interest rate, i_t^d.[10] Parameter σ_c is calibrated by setting the steady state spread between nominal retail loan interest rate, i_t^r, and nominal policy interest rate, i_t, at 7.1%.[11] The value of real collateral coefficient, κ^r, is calibrated by fixing the household loan GDP ratio, $\frac{\overline{\ell^c}}{\overline{gdp}}$, at 30%in the steady state.[12]

The AR(1) coefficient and standard deviation of productivity shock, $\rho_a = 0.95$ and $\sigma_a = 0.007$, follows Claus (2007) who develop a theoretical small open economy model to assess the effects of bank lending in New Zealand. With regard to the other exogenous shock parameters, we fit AR(1) model to log central bank policy rate in the United States (our proxy for foreign interest rate) and log GDP in the United States (our proxy for export demand) to obtain the estimates of ρ_{i^*}, σ_{i^*}, ρ_{exp} and σ_{exp}.[13]

[6] Source: New Zealand Ministry of Business, Innovation and Employment (MBIE).

[7] Source: New Zealand Ministry of Business, Innovation and Employment (MBIE) and New Zealand Treasury.

[8] We use the steady state assumption of Benes and Kumhof (2015) that 2% banks violate MCAR.

[9] The sum of minimum capital adequacy ratio of 8% and the capital conservation buffer of 2.5% give the actual capital adequacy ratio as 10.5%.

[10] Barrdear and Kumhof (2016) use a similar exercise to calibrate χ by implying the steady state spread between deposit rate and private mortgage rate. As our model is devoid of the mortgage sector, we calibrate χ on the basis of spread between wholesale lending rate and deposit rate. We use quarterly data on deposit and lending rate from *International Financial Statistics*. The average spread between deposit and lending rate from 2000:Q1 to 2007:Q4 in New Zealand is 1.5%.

[11] Barrdear and Kumhof (2016) calibrate σ_c by implying the steady state spread between retail interest rate and policy interest rate. We perform a similar exercise for New Zealand. We use monthly data on credit card lending rate (all type of credit cards) from *Reserve Bank of New Zealand* as a proxy for nominal retail loan interest rate. Monthly data on central bank policy interest rate of New Zealand is obtained from *International Financial Statistics*. We compute the monthly average of the spread between retail loan interest rate and policy interest rate from 2000:M1 to 2007:M12. Finally, we convert the average value of the spread to a quarterly rate.

[12] Barrdear and Kumhof (2016) calibrates κ^r of United States by implying the steady state consumption loan to GDP ratio. We use household debt as a proxy for household consumption loan. The average of household debt to GDP ratio of New Zealand from 1990:Q1 to 1994:Q4 equals 30%. Source of data: *Bank for International Settlements.*

[13] We fit AR(1) process on quarterly HP filtered US central bank policy interest rate data from 1955:Q1 to 2007:Q4 to estimate ρ_{i^*} and σ_{i^*}. Source of data: *Bank for International Settlements*. To estimate ρ_{exp} and σ_{exp}, we fit AR(1) process on quarterly HP filtered US real GDP data from 1955:Q1 to 2007:Q4. Source of data: *Bureau of Economic Analysis*.

3.10.2 Steady states

At the steady state, domestic good price, inflation, nominal exchange rate, real exchange rate and imported intermediate good price equal 1 so $\overline{P} = \overline{\pi} = \overline{e} = \overline{q} = \overline{f^{k*}} = 1$. Steady state marginal cost $\overline{\mathcal{MC}^r}$ equals to $\frac{\theta_p - 1}{\theta_p}$. The steady state exports-to-GDP ratio is set at 30% in line with the estimation of Jacob and Munro (2018). We restrict the CBDC-to-GDP ratio, $\overline{m^{rat}}$, at 20% in the steady state in order to obtain a positive spread between policy interest rate and CBDC interest rate. The steady states of all other variables are computed using a nonlinear solver in Matlab.

3.11 Welfare

Following Schmitt-Grohé and Uribe (2007), the welfare function is given by

$$\mathcal{W}_{j,t} = u_{j,t} + \beta_j E_t \mathcal{W}_{j,t+1}, j \in c, u \tag{61}$$

where $u_{j,t}$ is the utility function and j denotes whether the agent is the household or the financial investor. The weighted average of the welfare of the households and the financial investors is the economy's overall social welfare and given by.

$$\tilde{\mathcal{W}}_t \equiv (1-\omega)\mathcal{W}_{c,t} + \omega \mathcal{W}_{u,t} \tag{62}$$

To make comparisons between adjustable CBDC interest rate regimes[14] and the baseline regime, we use compensating consumption variation as in Kumhof and Laxton (2009). The compensating consumption variation is denoted by η_j^{price}, which is the percentage of consumption goods that agent j would be willing to forgo to remain indifferent between baseline regime and price rule regime. Following Kumhof and Laxton (2009), η_j^{price} is given by,

$$\eta_j^{price} = 100\left(1 - exp\left(\frac{(\beta_j - 1)\left(E\mathcal{W}_j^{price} - E\mathcal{W}_j^{baseline}\right)}{1-\upsilon}\right)\right), \; j \in c, u \tag{63}$$

In a similar manner, the equation below compares the welfare between the quantity rule regime and baseline regime.

$$\eta_j^{qty} = 100\left(1 - exp\left(\frac{(\beta_j - 1)\left(E\mathcal{W}_j^{qty} - E\mathcal{W}_j^{baseline}\right)}{1-\upsilon}\right)\right), \; j \in c, u \tag{64}$$

$E\mathcal{W}_j^{baseline}$, $E\mathcal{W}_j^{price}$ and $E\mathcal{W}_j^{qty}$ denote the unconditional expectation of welfare of agent j in the baseline, the

[14] Adjustable CBDC interest rate regimes correspond to price rule regime and quantity rule regime.

price rule and the quantity rule regimes, respectively. Following Bilbiie (2008) and Kumhof and Laxton (2013), the overall welfare gain of CBDC policy regime is the population weighted average of the compensating consumption variations. So we have,

$$\hat{\eta}^{price} \equiv (1-\omega)\eta_c^{price} + \omega\eta_u^{price}; \; \hat{\eta}^{qty} \equiv (1-\omega)\eta_c^{qty} + \omega\eta_u^{qty} \tag{65}$$

We use DYNARE to compute the unconditional welfare measures and compensating consumption variations through a second order approximation of the model.

4 Results

4.1 Optimal coefficient values

We assume that the central bank/government is a social planner whose objective is to maximise the overall welfare in the economy. To this purpose, we conduct grid searches for optimal policy coefficients that achieve the highest overall social welfare. In the simulations, the economy is subject to domestic productivity shock, foreign interest rate shock and export demand shock.

We find the optimum policy coefficient values of the interest rate rule (Equation 46) and fiscal policy rule (Equation 45) by conducting a five dimensional grid search for the coefficients ρ_i, $\varpi_{i,\pi}$, $\varpi_{i,q}$, $\varpi_{i,gdp}$ and $\varpi_{b,gdp}$ to achieve the highest level of social welfare, $\tilde{\mathcal{W}}$. We consider the ranges of 0 to 0.9 for the smoothing parameter, ρ_i, 0 to 3 for the responses of nominal interest rate to the real exchange rate and output, $\varpi_{i,q}$ and $\varpi_{i,gdp}$, respectively, 1.5 to 3 for the response of nominal interest rate to inflation, $\varpi_{i,\pi}$, and 0 to 3 for the response of government adjusted budget deficit ratio to GDP, $\varpi_{b,gdp}$.[15] We find the optimal coefficient values in the baseline regime at $\rho_i = 0$, $\varpi_{i,\pi} = 3$, $\varpi_{i,q} = 0$, $\varpi_{i,gdp} = 0$ and $\varpi_{b,gdp} = 3$.

Next, we optimise the coefficient associated with the CBDC price rule regime (Equation 48) where the nominal CBDC interest rate ($i_{m,t}$) acts as a secondary monetary policy instrument in addition to the primary policy interest rate rule (Equation 46). We keep the coefficients ρ_i, $\varpi_{i,\pi}$, $\varpi_{i,q}$, $\varpi_{i,gdp}$ and $\varpi_{b,gdp}$ at the optimised values as in the baseline regime and conduct a grid search for $\varpi_{i^m,\pi}$ to maximise the social welfare of the economy, $\tilde{\mathcal{W}}$. We take the range $[0, 8]$ as values of the price rule coefficient, $\varpi_{i^m,\pi}$[16] and $[0, 0.9]$ for smoothing parameter, ρ_{i^m}. We conduct a two dimensional grid search over the range $i_\pi^m \in [0, 8]$, $\rho_{i^m} \in [0, 0.9]$ and find the optimum coefficient at $\varpi_{i^m,\pi} = 8$, $\rho_{i^m} = 0.2$.

The final step of welfare optimisation is performed under the CBDC quantity rule regime (Equation 49) where

[15] We keep the lower limit of $\varpi_{i,\pi}$ grid search at 1.5 following Schmitt-Grohé and Uribe (2007) that equilibrium is indeterminate when $\varpi_{i,\pi} < 1$.

[16] Barrdear and Kumhof (2016) consider the range of $[0, 0.8]$ for the price rule coefficient. However, we find that the counter cyclical effects of price rule is very limited at 0.8. We keep the upper limit as 8 in line with range considered by Barrdear and Kumhof (2016) for quantity rule coefficient.

the CBDC-to-GDP ratio (m_t^{rat}) acts as the secondary monetary policy instrument instead of the nominal CBDC interest rate under the price rule regime (Equation 48). As in the optimisation in the CBDC price rule regime, we keep the optimal values of coefficients ρ_i, $\varpi_{i,\pi}$, $\varpi_{i,q}$, $\varpi_{i,gdp}$ and $\varpi_{b,gdp}$ as in the baseline regime and conduct a grid search for the optimal value of coefficient $\varpi_{m,\pi}$. Following Barrdear and Kumhof (2016), we keep the range of $\varpi_{m,\pi}$ in $[0, 8]$. We do a one dimensional grid search over the range $\varpi_{m,\pi} \in [0, 8]$ and find the optimum $\varpi_{m,\pi}$ at 8.

4.2 Welfare

We also consider the implications of the volatility of the instrument $i_{m,t}$, and social welfare gain through compensating consumption variation, $\hat{\eta}^{price}$, for a feasible CBDC price rule coefficient.[17] Hence, we plot the social welfare gain on the vertical axis and standard deviation (SD) on the horizontal axis of $i_{m,t}$ for the corresponding values of the coefficient $\varpi_{i^m,\pi}$ in Figure 3. Positive welfare gain values in the vertical axis indicate social welfare improvement when the economy moves from the baseline regime to the price rule regime for all values of coefficient $\varpi_{i^m,\pi}$. Figure 3 also shows that welfare gain increases rapidly as the SD of $i_{m,t}$ rises slowly when $\varpi_{i^m,\pi}$ increases between 0 to 1. However, for $\varpi_{i^m,\pi} > 1$, welfare gain rises at a decreasing rate relative to the rise in SD of $i_{m,t}$ since the welfare gain-SD curve becomes flatter. We also plot the social welfare gain, $\hat{\eta}^{qty}$, on the vertical axis and standard deviation (SD) of m_t^{rat} in the horizontal axis corresponding to the values of $\varpi_{m,\pi}$ in the range $[0, 8]$ in Figure 4. The figure shows a steeply upward sloping welfare gain-SD curve such that both social welfare gain and standard deviation (SD) of m_t^{rat} rises as $\varpi_{m,\pi}$ increases. The relatively steep slope of welfare gain-SD curve indicates that the the rate of increase of social welfare is higher than rate of increase in volatility (SD) of m_t^{rat} when $\varpi_{m,\pi}$ rises from 0 to 8.

A comparison between Figure 3 and Figure 4 reveals that quantity rule is accompanied by higher welfare gains and higher instrument volatility when compared to the price rule regime. Society's higher welfare gain in the quantity rule compared to the price rule is confirmed in Table 2 which shows the optimal policy coefficients and the associated welfare gains of an adjustable CBDC interest rate through price rule and quantity rule. However, a breakdown of the welfare estimates of households and financial investors in Table 2 shows that while households have positive welfare gain under the price rule and an even larger positive welfare gain under the quantity rule, financial investors have a small positive welfare gain under the price rule and a welfare loss under the quantity rule. This may be explained by the fact that households face monetary transactions costs in consumption spending (Equation 10) whereas financial investors do not. Given the monetary transaction costs, households would prefer to use interest bearing CBDC for consumption spending as it improves efficiency and lowers monetary transaction costs compared to deposits. To summarise, Table 2 shows that the welfare gain of households is highest under the

[17] Benes and Kumhof (2015) discuss the importance of considering both welfare gain and volatility of the policy instruments in determining the optimal coefficient values.

quantity rule regime with the CBDC-to-GDP ratio (m_t^{rat}) as the central bank's secondary policy instrument. The welfare loss of financial investors is also worst under the quantity rule regime. However, the large improvement in households' welfare more than compensates for the loss of financial investors' welfare so society's welfare gain is highest under quantity rule regime.

4.3 Contribution of individual shocks

Our grid search exercises described above aim to identify the policy responses that deliver the best welfare gains for the society, when all shocks are simultaneously present. It is possible that these optimal policies may deliver better welfare outcomes under certain shocks than the others. As such, it is helpful to examine the welfare gains of the productivity shock, the foreign interest rate shock and the export demand shock one at a time.

Table 3 displays the welfare gains for society, households, and financial investors when the economy is hit by one shock at a time. The table shows that the CBDC regimes with an adjustable interest rate deliver better welfare gains under the foreign shocks than domestic shock. Under the domestic productivity shock, households have a small welfare gain while financial investors have a small welfare loss when switching from a the baseline regime to adjustable-interest regimes. Under a foreign interest rate shock, households gain from the adjustable-interest regimes, more so under the quantity rule than the price rule. However, financial investors suffer a large welfare loss in the quantity rule regime with no improvement in welfare under the price rule regime. Society still has welfare gains under adjustable CBDC interest rates with the gain larger under the quantity rule than the price rule. Under a export demand shock, households have much larger welfare gains under the quantity rule than the price rule. Financial investors have a negligible welfare gain under price rule and a small welfare loss under the quantity rule regime. Hence, society has small welfare gain under the price rule and a large welfare gain under the quantity rule. Overall, society's welfare gains are found to be the largest under export demand shock. Society's higher welfare gains are also driven by higher household welfare in each of the shocks under the quantity rule regime but such welfare improvements in quantity rule regime is accompanied by larger welfare losses for financial investors.

4.4 Volatility

In addition to welfare gains, the volatility implications measured by the standard deviations of the key variables under the three regimes are shown in Table 4. In the simulation, the economy is subject to all the three shocks. Table 4 shows the standard deviation of GDP increases when the economy moves from the baseline regime to the adjustable CBDC interest bearing system of the price rule and the quantity rule. This is due to the impact on GDP from the larger changes to liquidity (both deposits and CBDC) in an adjustable CBDC interest rate regime. The standard deviation of consumption also increases when the economy moves from the baseline regime to the price rule but does not change under the quantity rule. In contrast, standard deviation of inflation declines under the price rule regime, effectively satisfying the price stability mandate of the central bank. The real exchange rate is also less volatile when CBDC interest rate is adjustable with the decline the largest under the quantity rule regime.

4.5 Impulse responses

In this section, we discuss the impulse responses under the three regimes following a domestic productivity shock, a foreign interest rate shock and a export demand shock in the economy.

Productivity shock: Figure 5 shows the impulse responses of key macroeconomic variables following a 0.7 basis points productivity shock. The productivity shock increases the supply of domestic (consumption) goods, which in turn initially causes lower inflation. With inflation lower than its target, the nominal policy interest rate (i_t) declines. The nominal policy interest rate drops by more in the baseline regime than in the regimes with CBDC as a secondary policy instrument. The drop in the nominal interest rate makes government bonds less attractive to financial investors.

With the CBDC as secondary policy instrument, the CBDC interest rate rises when inflation is below its target under the price rule regime while the CBDC-to-GDP ratio (m_t^{rat}) increases when inflation is below its target under the quantity rule. In the price rule regime, CBDCs with higher interest become more attractive to households who require liquidity as consumption rises while government bonds are less attractive to financial investors as nominal interest rate drops. To meet the changing demands of households and financial investors, the central bank increases CBDCs and reduces government bonds. This endogenously increases the liquidity (refer to subsection 3.6.2). In contrast, under the CBDC quantity rule, the central bank increases the supply of CBDC (in exchange of government bonds) as inflation is less than target inflation so the CBDC become less valuable and households are only willing to hold CBDCs at higher CBDC interest rate (refer to subsection 3.6.3). Under all regimes, government bonds decline. In addition, the increase in CBDCs means increased efficiency, lower financial transaction cost and lower "liquidity tax." The impulse responses of the CBDC-to-GDP ratio reflect the lower volatility of the CBDC price rule than the CBDC quantity rule.

Following the domestic productivity shock, the ensuing higher demand for liquidity is further enhanced by additional bank loans that create additional bank deposits. Figure 5 shows that aggregate deposits and loan rise in all the three regimes. The endogenous rise in liquidity strengthens the rise in real economic activity. Higher demand for bank loans and deposits fuels a rise in the nominal deposit and wholesale lending rate. The path of nominal deposit and wholesale lending interest rates follow the rise of the nominal CBDC interest rate.

Since foreign interest rate is exogenous, the decline in the nominal policy interest rate with uncovered interest parity causes the exchange rate to depreciate. The depreciation causes marginal cost of imported intermediate inputs to rise. Eventually, the marginal cost of imported inputs rises more than the drop in marginal cost due to higher productivity. Hence, inflation moves from a large negative inflation to a small positive inflation in the second period. However, we note the uninterrupted adjustment of nominal policy interest rate to their steady state values under both the CBDC price rule and the CBDC quantity rule. During the second period, the CBDC-to-GDP ratio and nominal CBDC interest rate of adjustable CBDC interest rate regimes decline by more than their steady state values. During the same period, bank deposits and bank loans fall slightly below their steady state. Following the fall in bank loans and deposits, consumption drops below the steady state before moving back to the steady state. In moving toward its steady state, consumption declines by more under the baseline regime than the adjustable CBDC rate regimes.

The exchange rate response is similar across all the regimes. The depreciation of the exchange rate causes the exchange rate adjusted cost of foreign borrowing to rise, so foreign debt initially declines in all the regimes. As foreign debt falls more than the real exchange rate depreciation, risk premium rises (Equation 37). The fall in foreign debt together with the fall in imports causes the balance of payments to rise.

The impulse responses due to domestic productivity show minimal differences between adjustable CBDC interest-bearing price rule and quantity rule regimes. Barrdear and Kumhof (2016) also note the negligible differences in the CBDC regimes after a productivity shock. There is noticeable, albeit, small differences between fixed CBDC interest and adjustable CBDC interest-bearing regimes. In contrast, the differences become more pronounced when we consider small open economy specific shocks like foreign interest rate shock and export demand shock detailed in the sections below.

Foreign interest rate shock: Figure 6 shows the impulse responses of the key variables following a positive foreign interest rate shock (24.37 basis points increase in the foreign interest rate). The rise in foreign interest rate depreciates the exchange rate through the uncovered interest parity condition. Firms now face higher marginal cost as the imported intermediate goods for production becomes more expensive. This has a feedback effect on the domestic good prices, causing inflation to rise in the economy. The baseline regime witnesses more volatile inflation when compared to the adjustable CBDC interest rate regimes. With the rise in marginal cost and domestic prices, quantity demand for consumption goods initially declines.

In light of higher inflation, the monetary policy instruments become active to help the economy move back to price stability. Nominal policy interest rate rises in all the three regimes with baseline regime witnessing the highest rise. The higher inflation also reduces the nominal CBDC interest rate for the price rule regime (through the downward adjustment in the CBDC interest rate itself) and the quantity rule regime (through downward adjustment in the CBDC-to-GDP ratio). Hence, the CBDC-to-GDP ratio declines, with the decline more significant under the price rule regime.

The decline in consumption demand causes the demand for liquidity to shrink in the economy. Hence, we see that the aggregate bank deposits decline in Figure 6. As a result, the liability exposure of banks in terms of deposits also decline. This indirectly creates a reduction in the bank riskiness, which has a feedback effect on the nominal wholesale lending rate via Equation 7. The wholesale lending rate declines more than the deposit rate following the shock. In the adjustable CBDC interest rate regimes, the response of the nominal deposit rate mimics the behaviour of nominal CBDC rate.

The exchange rate depreciation is also found to be more volatile in the baseline regime. The rise in the exchange rate adjusted cost of borrowing causes the foreign debt to decline in all the regimes. As foreign borrowing by financial investors (foreign debt) declines, the risk premium moves downward. We also note as foreign debt and risk premium together with imports decline so the balance of payment improves.

Export demand shock: The impulse responses of the key variables following a export demand shock are shown in Figure 7. The exogenous rise in exports increases aggregated demand and domestic prices. The higher prices and inflation cause the first monetary policy instrument of nominal policy interest rate to initially rise under all the three regimes. The rise in nominal policy interest rate is slightly higher under the quantity rule regime following the export demand shock. The higher inflation also causes either the nominal CBDC interest rate to fall under the price rule regime (with the corresponding fall in CBDC-to-GDP ratio) or the CBDC-to-GDP ratio in the quantity rule to decline under the quantity rule regime (with the corresponding decline in the CBDC interest rate).

The higher prices lower the quantity consumption demand and reduce the demand for liquidity in the economy. Hence, both deposits and loan decline as well. The decline in CBDCs and deposits, and the rise in consumption liquidity tax are stronger under the quantity rule regime than either the price rule regime or the baseline regime. The strong rise in liquidity tax escalates the monetary transaction cost of using deposit, which increases the expected bank riskiness.[18] This results in the rise of wholesale lending rate following the shock. In contrast, the nominal deposit rate mimics the drop in the nominal CBDC interest rate. The decline in the nominal deposit rate is the least under the baseline regime followed by the price rule regime and the quantity rule regime.

Under the adjustable CBDC interest rate regimes, higher export increases trade which initially appreciates the exchange rate. The exchange rate appreciation also lowers the cost of foreign borrowing so foreign debt and the risk premium initially rise.

After the initial jump in inflation following the shock, inflation starts to decline in the second period causing the nominal policy rate to decline as well. However, the nominal CBDC interest rate and the CBDC-to-GDP ratio continue the smooth adjustment to the steady state. The decline in the nominal policy interest rate causes the real exchange rate to eventually depreciate, with the depreciation the largest under the baseline regime followed by the price rule regime and then the quantity rule regime. Hence, foreign debt and the risk premium also decline. The balance of payment becomes positive as exports rise while imports, foreign debt and risk premium drop.

[18]The rise in consumption liquidity tax is significantly larger with an export demand shock when compared to foreign interest rate shock. Although liability exposure in terms of deposits decline, the strong rise in consumption liquidity tax increases the expected bank riskiness.

5 Sensitivity analysis

As discussed in subsection 3.7, the elasticity of substitution between CBDCs and deposits ($\frac{1}{1-\theta}$) has an important bearing on the effectiveness of the CBDCs as secondary monetary policy instruments. We explore this by performing sensitivity analyses on the economy's responses following an export demand shock by varying the value of θ. We perform the sensitivity analyses using the export demand shock as the CBDC monetary policy instruments are found to be the most welfare improving following an export demand shock (see subsection 4.2).

Figure 8 shows the impulse responses for θ equals 0.65, 0.95 and 1 for the CBDC price rule regime with all other calibration parameters remaining the same as in subsection 3.10 and policy coefficient values as in Table 2. We find that deposits, nominal CBDC rate and nominal deposit rate decline following an export demand shock. The figure shows larger declines with low substitutability ($\theta = 0.65$) than high substitutability ($\theta = 0.95$, $\theta = 1$). When CBDCs and deposits are perfect substitutes ($\theta = 1$), the spread between the deposit and CBDC rate is fixed. A lower substitutability between CBDCs and deposits ($\theta = 0.65$) makes the CBDC rate more independent from the deposit rate (as explained in Equation 51), causing the spread between the deposit rate and CBDC rate to decline significantly. We can see from Figure 8 that the countercyclical behaviour of the CBDC rate is stronger for lower values of θ. Liquidity in terms of CBDC-to-GDP ratio also declines more significantly when $\theta = 0.65$. With lower substitutability between CBDCs and deposits, the consumption liquidity taxes increase by less with corresponding effects on real economic activity.

In a similar manner, Figure 9 shows the impulse responses for θ equals 0.65, 0.95 and 1 for the CBDC quantity rule regime. Deposits and the nominal deposit rate behave very differently from the nominal CBDC interest rate for lower substitutability between CBDCs and deposits ($\theta = 0.65$). This indicates greater independence of nominal CBDC rate from the deposit rate. The spread between CBDC and deposit rates also declines the furthest when $\theta = 0.65$. The CBDC-to-GDP ratio also declines, but only slightly lower, when $\theta = 0.65$. Consumption liquidity taxes increase much less with low than high substitutability between CBDCs and deposits.

Our results are consistent with Barrdear and Kumhof (2016), who note that the decline in deposits results in a stronger decline of the ”liquidity benefits” when CBDC and deposits are highly imperfect substitutes in a CBDC quantity rule regime.[19] Hence, any decrease in bank deposits strongly decreases the ”liquidity benefits” of CBDC for lower values of θ. Since under the CBDC quantity rule, the central bank fixes the quantity of CBDCs consistent with GDP growth and inflation, household would require a higher CBDC interest rate to hold CBDCs with lower liquidity benefits. Hence, we see in Figure 9 that the nominal CBDC rate rises above the steady state when $\theta = 0.65$. In contrast, under the price rule in Figure 8, the central bank fixes the nominal CBDC interest rate and allows the CBDC-to-GDP ratio to drop. Hence, the CBDC-to-GDP ratio drops by a lot more with low substitutability of CBDC and deposits to reflect the lower liquidity benefits of CBDCs.

[19] Barrdear and Kumhof (2016) discuss the non-pecuniary benefits of holding the limited quantity of CBDC when deposits and CBDC are imperfect substitutes. A decline in deposits reduces such non-pecuniary benefits of holding CBDC.

6 Conclusion

While the idea of CBDCs is not new, breakthroughs in technology in the use of digital currencies have made central banks seriously consider the introduction of CBDCs. Our study contributes to the discussion on the benefits of CBDCs and its policy implications. We find that CBDCs could be an effective secondary monetary policy instrument for maintaining price stability in a small open economy. We show that society's welfare in an economy with adjustable CBDC interest rate regimes using the price rule and the quantity rule is higher than a regime with fixed CBDC interest rate. In an open economy, society's welfare gains are higher in the presence of foreign shocks (foreign interest rate and export demand shocks) than domestic (productivity) shock. With foreign shocks, society's welfare gains are highest when the primary interest rate rule is supplemented with the secondary CBDC quantity rule. However, in a heterogeneous agent framework and with adjustable CBDC interest rate regime, we find that those with access to CBDCs (households) are made better off while those without access to CBCDs (financial investors) are made worse off when there are shocks to the economy.

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Table 1: Parameter values

Symbol	Description	Value	Source
A_c	Monetary transaction cost function parameter	0.0279	Bacchetta and Perazzi (2018)
B_c	Monetary transaction cost function parameter	0.0241	Bacchetta and Perazzi (2018)
α	Share of labour in production function	0.7000	Authors' choice
η	Labour supply elasticity	1/3	Galí and Monacelli (2005)
ϕ	Fraction of firms with prices unchanged	0·7500	Galí and Monacelli (2005)
θ_p	Elasticity of substitution between differentiated goods	6.0000	Galí and Monacelli (2005)
β_u	Discount factor (financial investors)	0.9975	Benes et al. (2015)
β_c	Discount factor (households)	0.9809	Authors' estimate
ψ_f	Utility weight of liquidity	0.3526	Lozej et al. (2018)
ψ_h	Utility weight of hours worked	1.9282	Lozej et al. (2018)
ϕ_b	Financial asset transaction cost coefficient	0.0200	Lindé (2001)
ϑ	Elasticity of financial investor deposit demand	0.0125	Jacob and Munro (2018)
s_g	Govt spending to GDP ratio	0.1800	Jacob and Munro (2018)
υ	Consumption habit	0.7500	Jacob and Munro (2018)
φ_e	Risk premium coefficient	0.1000	Benes et al. (2015)
φ_c	Loan adjustment cost coefficient	0.0100	Benes et al. (2015)
τ_c	Consumption tax rate	0.1500	Authors' estimate
τ_L	Labour income tax rate	0.3000	Authors' estimate
ω	Proportion of financial investors in total population	0.0500	Barrdear and Kumhof (2016)
ι	Share of lump sum income received by households	0.9629	Barrdear and Kumhof (2016)
ς	Share of monitoring costs and monetary transaction cost in the goods market clearing	0.2500	Barrdear and Kumhof (2016)
κ^f	Financial collateral coefficient	1.0000	Barrdear and Kumhof (2016)
θ	Liquidity generating function parameter	0.9500	Barrdear and Kumhof (2016)
T^{fintec}	Technical efficiency coefficient of CBDC	1.1530	Barrdear and Kumhof (2016)
ξ	Bankruptcy cost parameter	0.0563	Barrdear and Kumhof (2016)
σ_b	Bank riskiness parameter	0.0100	Authors' estimate
Υ	Minimum Capital Adequacy Ratio	0.0800	Bank for International Settlements
δ^b	Share of bank networth paid as dividends	0.1727	Authors' estimate
ζ^c	Risk weight of consumption loan	0.7500	Bank for International Settlements
χ	Penalty coefficient for breaching MCAR	0.0150	Authors' estimate
σ_c	Collateral asset value riskiness parameter	1.0843	Authors' estimate
κ^r	Real collateral coefficient	0.9500	Authors' estimate
ρ_a	AR(1) productivity shock	0.9500	Claus (2007)
ρ_{r*}	AR(1) foreign interest rate shock	0.6788	Authors' estimate
ρ_{exp}	AR(1) export demand shock	0.8420	Authors' estimate
σ_a	Standard deviation of productivity shock	0.0070	Claus (2007)
σ_{r*}	Standard deviation of foreign interest rate shock	0.2437	Authors' estimate
σ_{exp}	Standard deviation of export demand shock	0.0080	Authors' estimate

Table 2: Coefficients of optimal policies

	ρ_i	$\varpi_{i,\pi}$	$\varpi_{i,q}$	$\varpi_{i,gdp}$	$\varpi_{b,gdp}$	ρ_{i^m}	$\varpi_{i^m,\pi}$	$\varpi_{m,\pi}$	Welfare gain		
									Society($\hat{\eta}$)	Household(η_c)	Financial Investor(η_u)
Baseline	0.9	1.5	0	0	3	-	-	-	-	-	-
Price rule	0.9	1.5	0	0	3	0.2	8	-	0.0284	0.0299	0.0008
Quantity rule	0.9	1.5	0	0	3	-	-	8	0.3855	0.4065	-0.0143

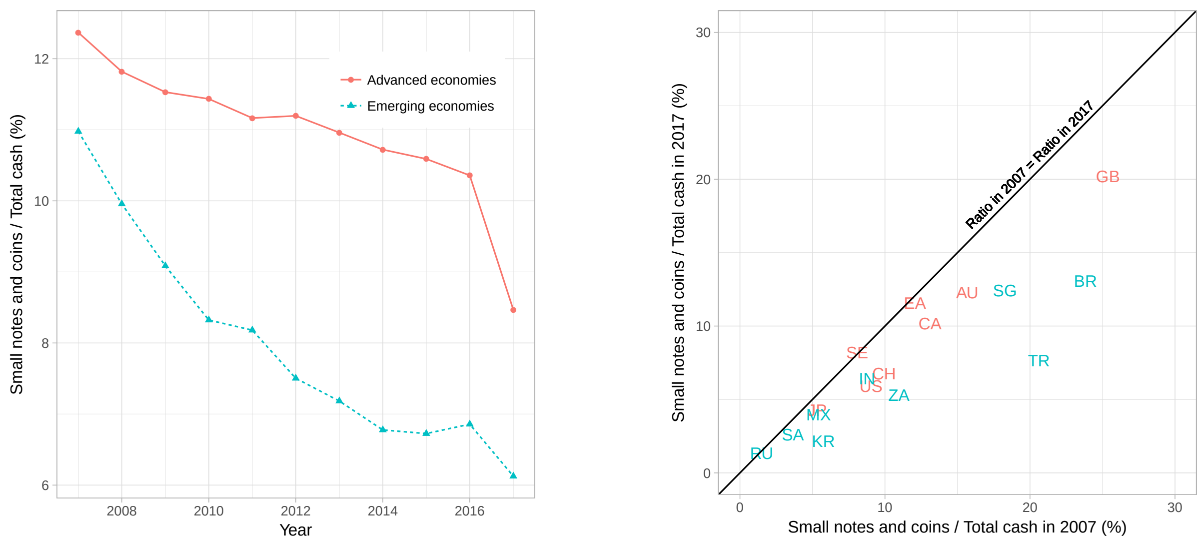
Table 3: Welfare gains and sources of shocks

	Productivity shock			Foreign interest rate shock			Foreign demand shock		
	Society($\hat{\eta}$)	Household(η_c)	Fin. inv.(η_u)	Society($\hat{\eta}$)	Household(η_c)	Fin. inv.(η_u)	Society($\hat{\eta}$)	Household(η_c)	Fin. inv.(η_u)
Baseline	-	-	-	-	-	-	-	-	-
Price rule	0.0004	0.0004	-0.0001	0.0017	0.0018	0.0000	0.0271	0.0285	0.0008
Quantity rule	0.0004	0.0004	-0.0002	0.0184	0.0194	-0.9063	0.3674	0.3875	-0.0134

Table 4: Volatility of key variables

	Baseline	Price Rule	Quantity Rule
GDP	0.0031	0.0032	0.0032
Consumption	0.0133	0.0134	0.0133
Inflation	0.0012	0.0010	0.0012
Real exchange rate	0.0765	0.0758	0.0752

Note: This table reports the standard deviations of key variables obtained from a stochastic simulation of the models using second-order perturbations around the stochastic steady state in Dynare 4.5.3. Variables are expressed in percentage deviations from the steady states.



(a) Average ratios by country groups.

(b) Change from 2007 to 2017

Figure 1: **Declining usage of small-denomination cash for transactions (2007-2017).** Small denominations are defined to be denominations smaller than that of the notes available at ATMs. These small denominations are usually used for daily transactions. Declining usages of this group of cash imply increasing adoption of electronic payment facilities. Countries in the advanced economies sample include Australia(AU), Canada(CA), Japan(JP), Sweden(SE), Switzerland(CH), United Kingdom(GB) and United States(US). The emerging economies sample includes Brazil(BR), India(IN), Mexico(MX), Russia(RU), Saudi Arabia(SA), Singapore(SG), South Africa(ZA), South Korea(KR) and Turkey(TR). Change in part (b) reflects from 2012 to 2017 for Euro Area and from 2007 to 2016 for United Kingdom.
Source: Bank for International Settlements.

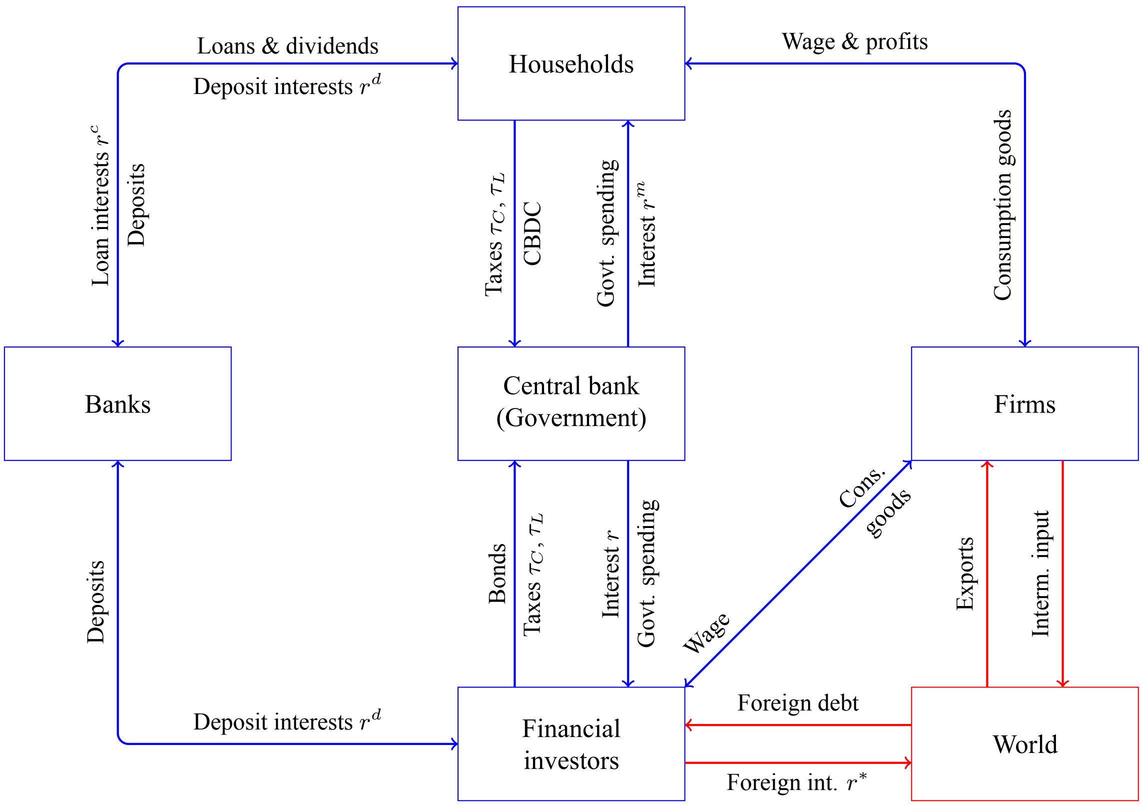


Figure 2: **Model outline.** Arrows indicate directions of fund flows between sectors.

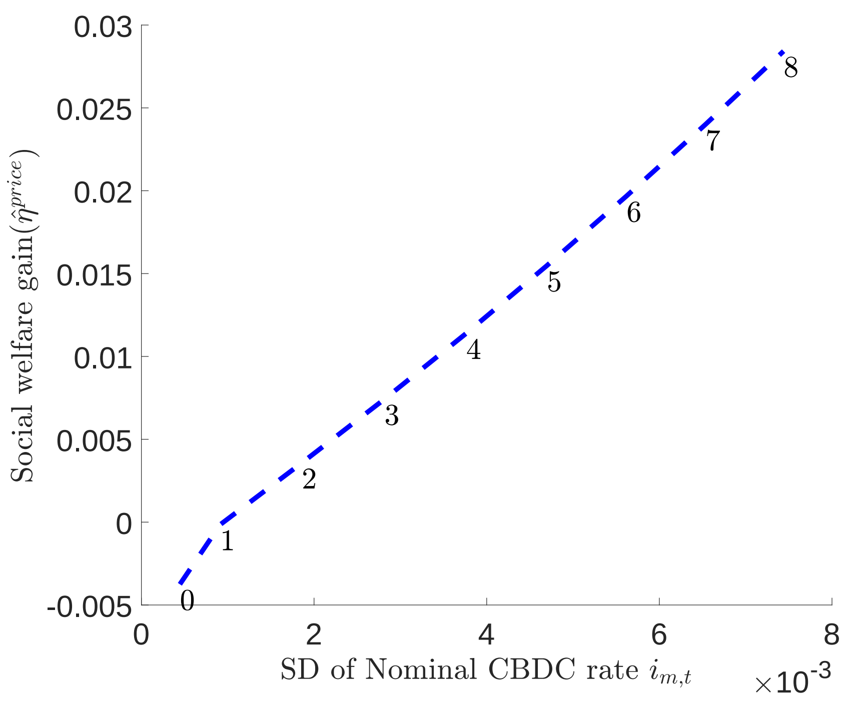


Figure 3: **Price rule regime welfare gain-SD curve.** Data labels denote the inflation feedback coefficient in price rule, $\varpi_{i^m,\pi} \in [0, 8]$.

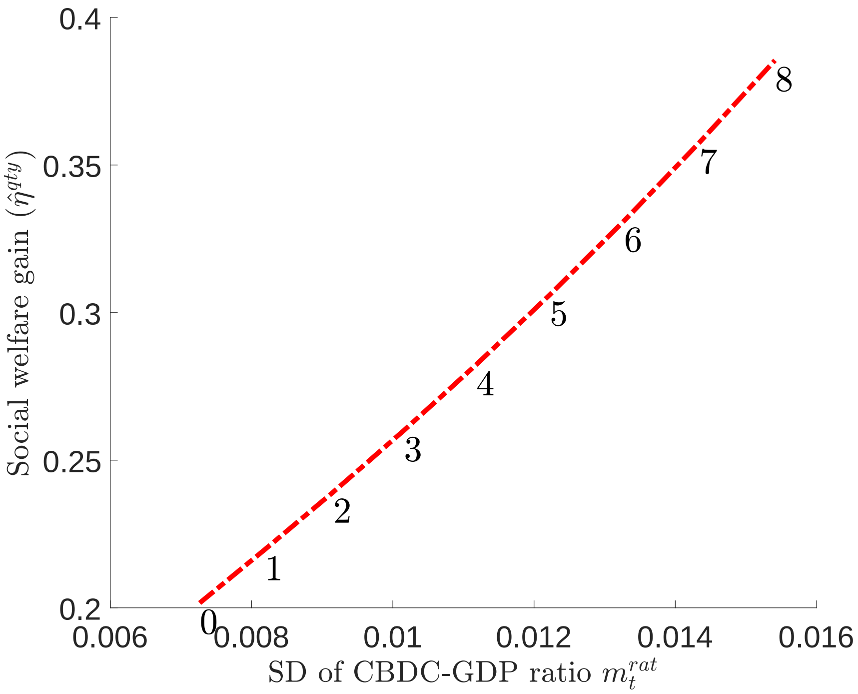


Figure 4: **Quantity rule regime welfare gain-SD curve.** Data labels denote the inflation feedback coefficient in quantity rule, $m_\pi \in [0, 8]$

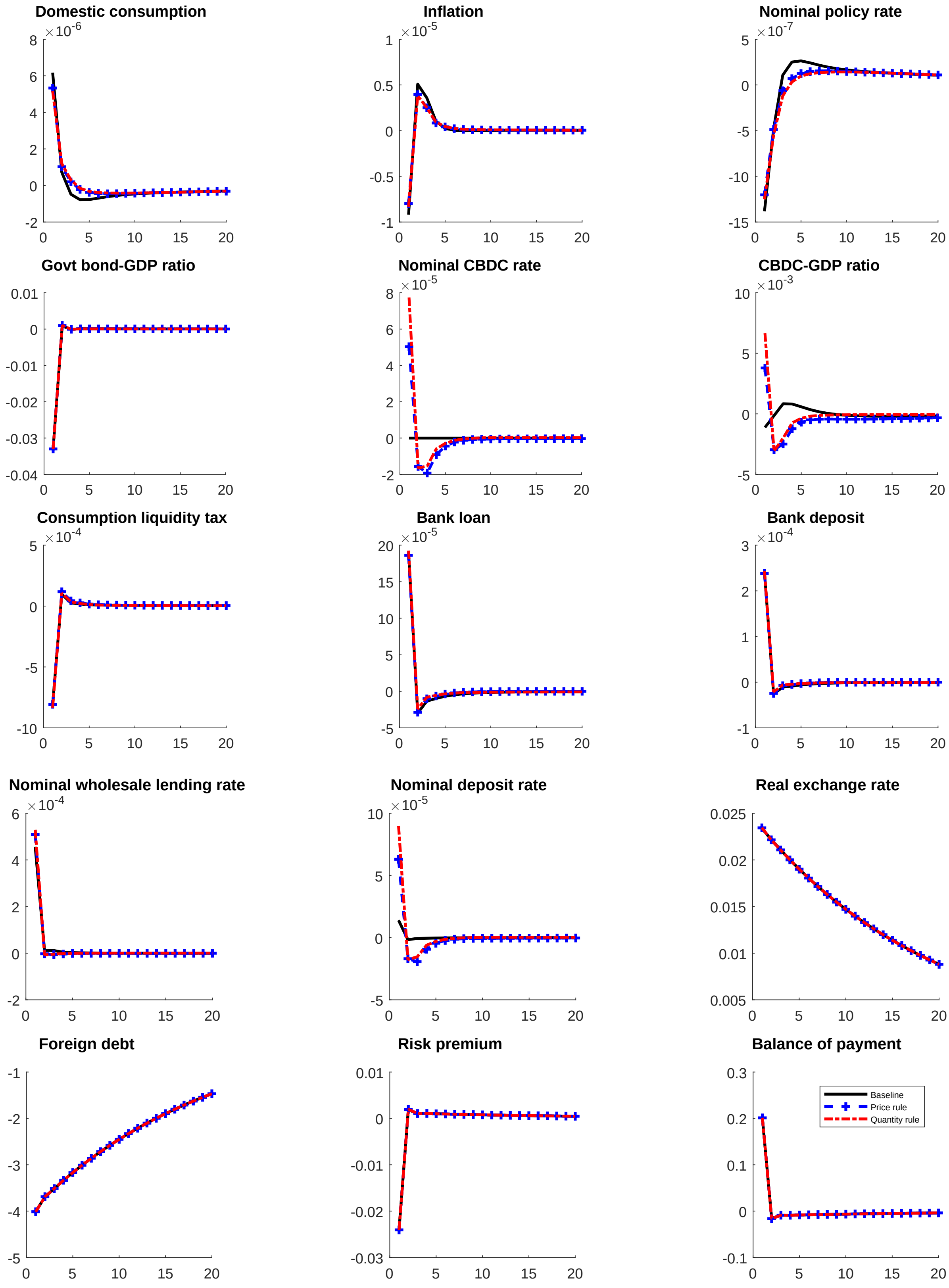


Figure 5: **Productivity shock**: Impulse response to a 0.7 basis point increase in productivity.

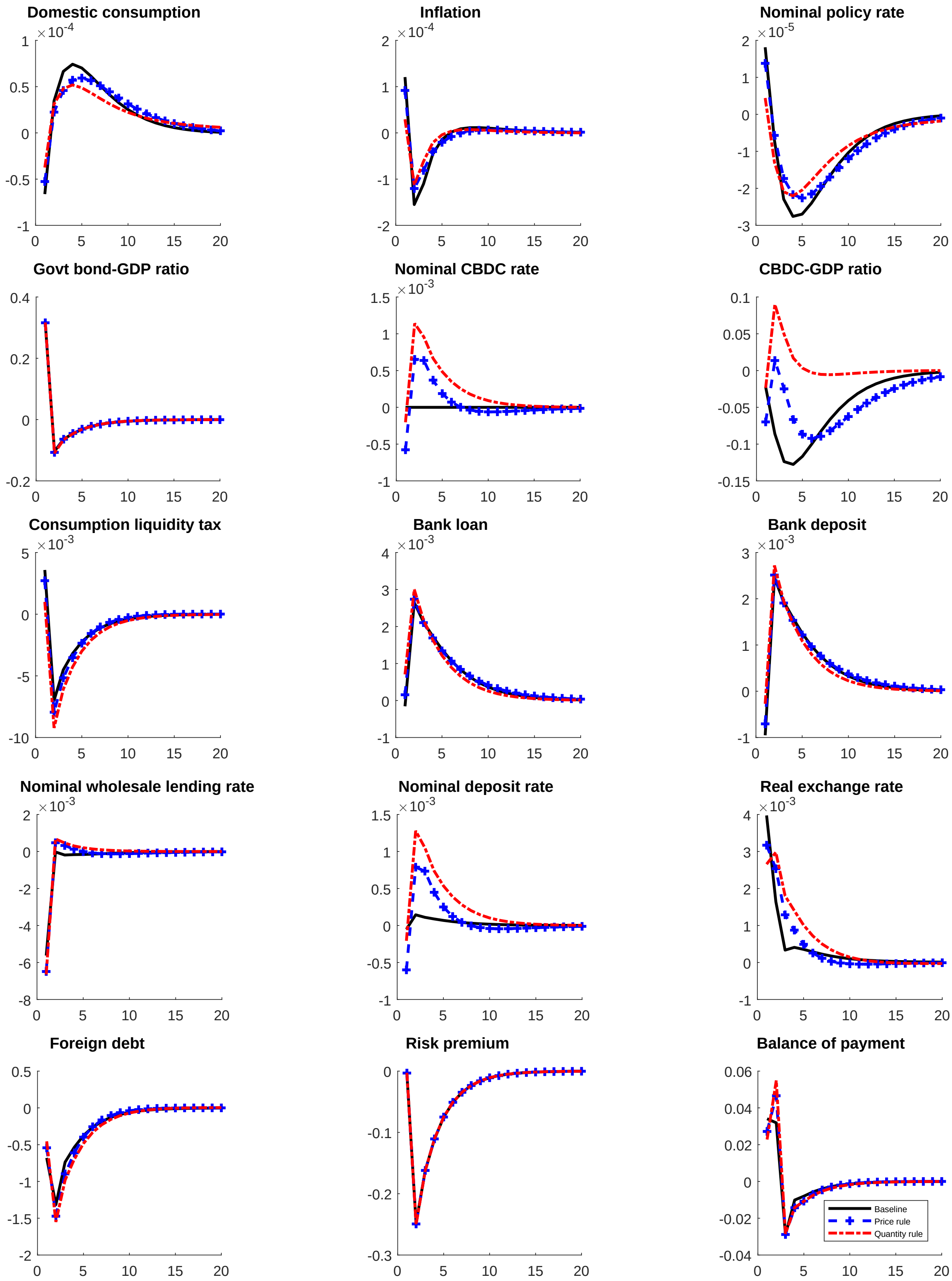


Figure 6: **Foreign interest rate shock**: Impulse response to a 24.37 basis point increase in foreign interest rate

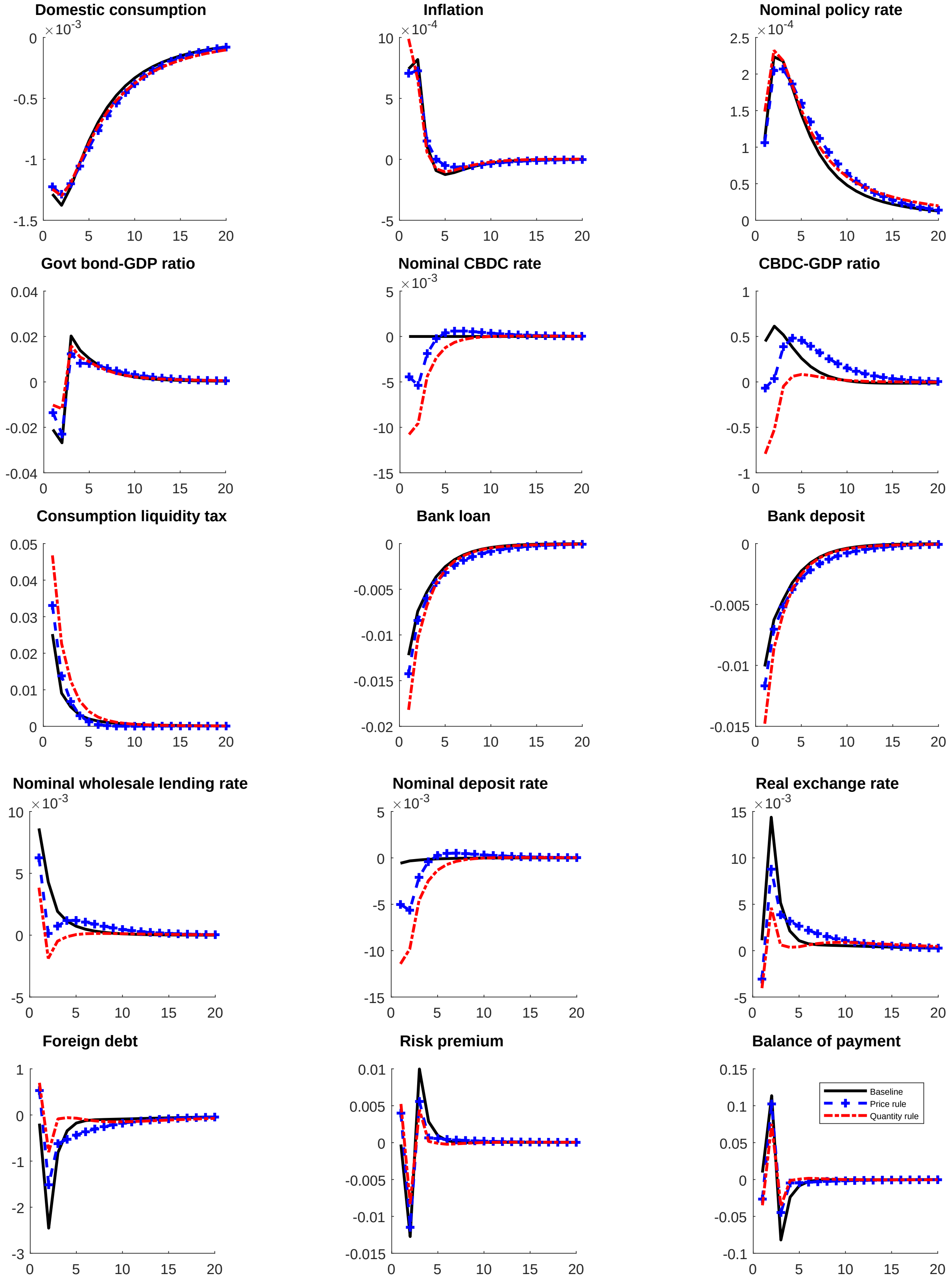


Figure 7: **Export demand shock**: Impulse response to a 0.8 basis point increase in export demand

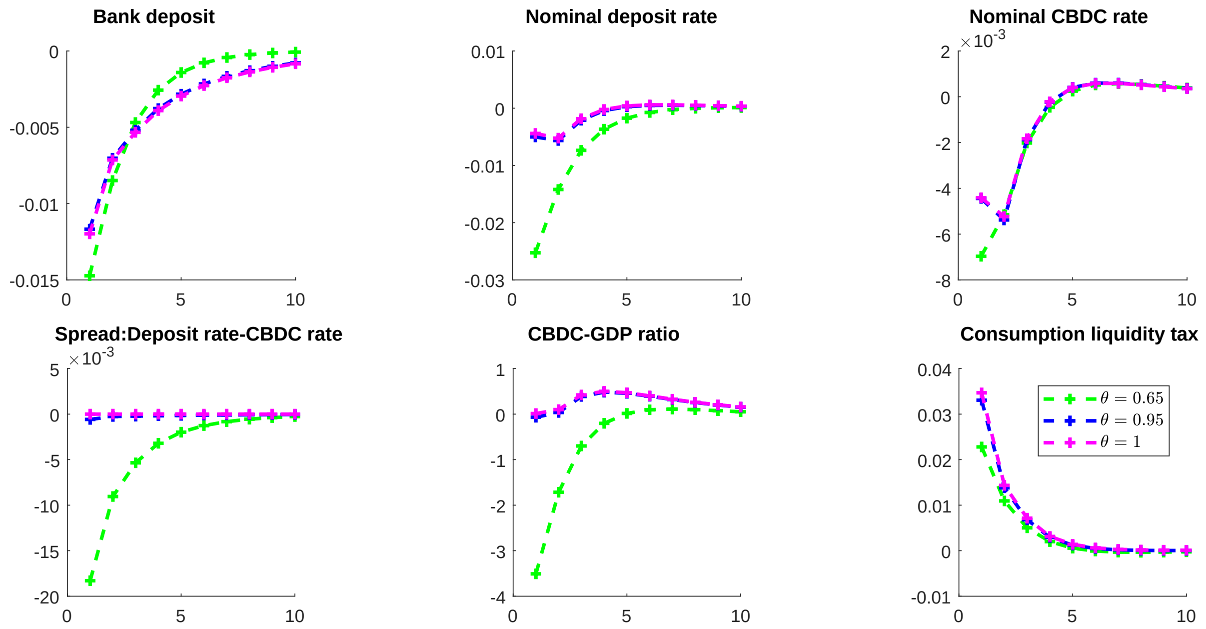


Figure 8: **Effect of varying θ in price rule regime**: Impulse response to a 0.8 basis point increase in export demand

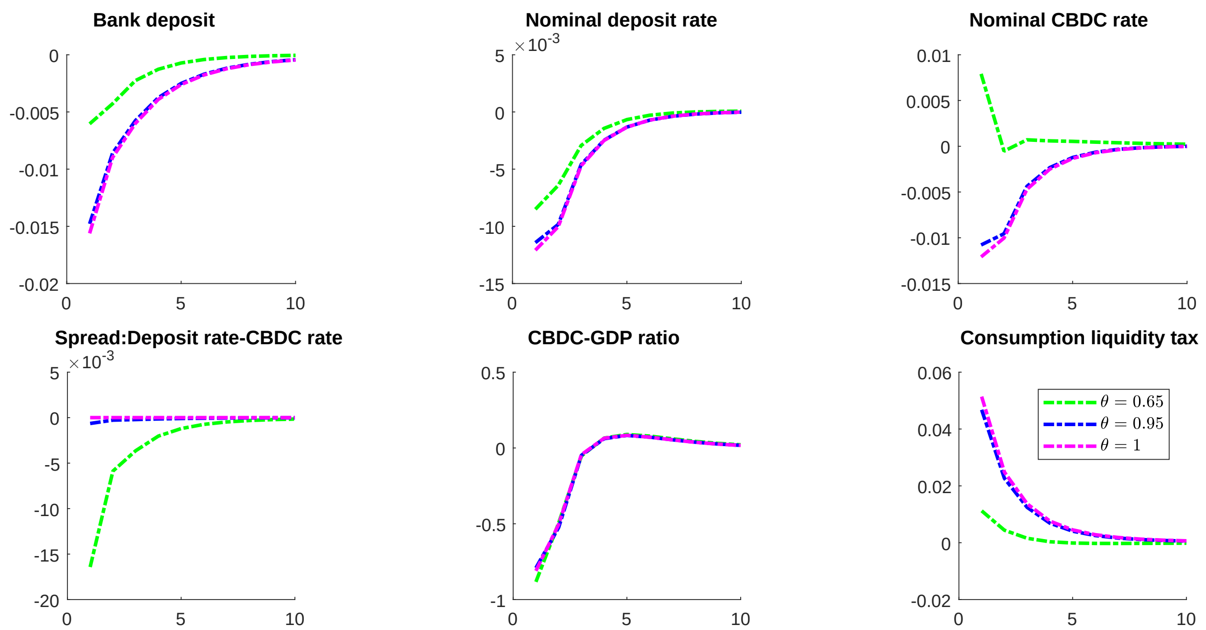


Figure 9: **Effect of varying θ in quantity rule regime**: Impulse response to a 0.8 basis point increase in export demand