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A conceptual model for FRAND royalty setting*

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Abstract

Setting an industry-wide standard is crucial for the interoperability, compatibility, and efficiency of information and communication technologies. To minimize holdup problems, patent holders are often required to ex-ante commit to licensing their technologies under Fair, Reasonable, and Non-Discriminatory (FRAND) terms. Yet, there is little consensus, in both courtrooms and industries, on the exact meaning of FRAND. We propose a new framework that enables a precise distinction: *fairness* in the distribution of royalty payments among patent users, and *reasonableness* in setting the size of the compensation to the patent holder, where both the size and the distribution of payments are determined in a *non-discriminatory* way that ensures similar firms are treated similarly.

JEL Classification: D63, K2, L3, L44

Keywords: FRAND-licensing, Fair royalties, Standard setting, Patent, Shapley value

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1 Introduction

Setting an industry-wide standard is crucial for the interoperability, compatibility and efficiency of information and communication technologies. However, once the standard has been set, a serious hold-up problem arises, as the patent holders now have substantially more bargaining power over licensing terms. To avoid opportunistic behavior, many Standard Setting Organizations (SSO's) require patent holders to commit to licensing their technologies under Fair, Reasonable, and Non-Discriminatory (FRAND) terms (Lerner and Tirole, 2014).[1]

Yet, the exact meaning of FRAND is ambiguous (European Commission, 2016) and various methods have been used with different results (Geradin, 2013). Because of the vague notion of FRAND, there have been much controversies in both courtrooms and industries regarding licensing terms. Moreover, the literature often uses FRAND and RAND (Reasonable and Non-Discriminatory) interchangeably, with some authors arguing that there is no distinction between them (e.g., Carlton and Shampine 2013).

A precise understanding of FRAND is important both for academic discussion and for real-world application in courtrooms. For instance, the patent *hold-up* problem can be seen as the result of vaguely defined licensing terms. Under the FRAND commitment, any patent user is only bounded by the FRAND royalty. If the upper-bound of FRAND is unclear, users may deliberately choose not to seek the license and exploit the legal uncertainty in court. This seriously reduces the incentive to innovate. Another benefit from a more precisely defined notion of FRAND is that it addresses *royalty stacking*. Without a precise definition of FRAND, patent holders of a standard can reasonably ask for the incremental value of their technologies. In this case, the sum of royalty payments usually exceeds the economically viable value of the standard and thereby defeats the purpose of standard setting.[2]

In this paper, we suggest a conceptual model for a FRAND royalty setting, embedding

[1] The notion of (F)RAND originates from different Standard Setting Organizations (SSO's), such as the European Telecommunications Standard Institute (ETSI) and the Institute of Electrical and Electronics Engineers (IEEE). Note that FRAND is commonly used in Europe and RAND is used in the US. See the further discussion on the history of (F)RAND in Carlton and Shampine (2013) and Ménière et al. (2015).

[2] The sum of all incremental values for all technologies exceeds the total value of the standard when technologies are complementary, which is usually the case for the standard setting.

a formal definition of FRAND within a framework of welfare economics. The conceptual model is meant to be a benchmark for practice. In particular, we propose an axiomatic approach linked to the literature on fairness in public goods provision that integrates a new version of earlier ideas that use the Shapley value to determine fair royalty fees. The model enables a precise distinction between *fairness* in the distribution of royalty payments among patent users and *reasonableness* in setting the size of the compensation for the patent holder. In the model, both the size and the distribution of payments are determined in a *non-discriminatory* way that ensures similar firms are treated similarly.

In particular, we consider a scenario where the industry, represented by an SSO, pools their patents (technologies). A subset of these technologies are chosen by the SSO to form an industry standard. Since the standard affects every firm in the industry, a reasonable compensation to each patent holder is based on the incremental contribution of the patent to the industry as a whole.[3] The reasonable compensation to each patent holder should then be fairly divided among all the firms of the industry (including the patent holders themselves). To set a fair royalty payment, we suggest proportional sharing relative to a given firm-specific liability index for each individual patent. This liability index is based on the firm-specific incremental benefits from access to various subsets of the available patents. For one example of a compelling liability index, we suggest using the Shapley value of the naturally induced cooperative game in patents for each firm in the industry. As such, our paper intends to formally capture fairness in the distribution of royalty payments through a mixture of proportionality with respect to a firm-specific characteristic — i.e., the firm's liability index for each patent — and Shapley's idea of fairness as the average incremental contribution of each available patent for each firm (Shapley, 1953).

To illustrate the conceptual framework in standard setting licensing, we apply our ap-

[3] We note that Layne-Farrar et al. (2007) suggested considering all marginal values related to the induced cooperative game (R, v), and then use the Shapley value with respect to (R, v) to allocate the total worth $v(R)$ among the individual technologies in R. The Shapley value can be interpreted as the compensation that the respective technology owners should receive. However, this type of direct approach has been criticized in the literature (see. e.g., Sidak 2013), since some firms may be rewarded even though their technology is worthless for the industry as whole in case of substitutes, for instance (simply because the marginal value of adding these technologies to some sub-coalition may be positive). Our approach obviously avoids this issue.

proach to several well-known models in industrial organization. First, we consider horizontal markets where every firm engages in Cournot competition in the same market (homogeneous products) or related markets (heterogeneous products).[4] When firms are symmetric, our FRAND liability implies equal sharing, i.e., firms pay identical royalty fees. Sidak (2013) mentions two rules: (a) *top-down* rule that equates per unit royalty to the product of the profit margin and the fraction of incremental contribution of the patent to the value of the standard, and (b) *proportional contribution* rule that equates per unit royalty to the product of the price of the final product and the fraction of incremental contribution of the patent to the value of the standard. Our approach coincides with the top-down rule and hence differs from the proportional contribution rule by a factor of relative markup. When firms are not symmetric, our approach is very different from the two rules since they are related to the total profit ratio while ours is related to marginal profits. Therefore, our approach coincides with the top-down rule only when all technologies are perfect complements. Moreover, when firms produce heterogeneous goods, our FRAND royalty depends on the market structure and firm characteristics. Our rule leads to equal sharing only when both firms and demand are symmetric.

Second, we consider vertical markets where upstream firms are indispensable to value creation and downstream firms engage in Cournot competition in the same market. FRAND royalties depend on the market structure: more upstream firms reduce the FRAND royalty of all firms; while more downstream firms will increase the royalty of upstream firms, but reduce the royalty of downstream firms.

Related Literature: The notion of (F)RAND has received considerable attention in the literature (e.g. Swanson and Baumol 2005; Layne-Farrar et al. 2007; Sidak 2013; Carlton and Shampine 2013; Leonard and Lopez 2014; Baron and Schmidt 2016) and from the US and EU competition authorities (e.g. U.S. Department of Justice and Patent & Trademark Office 2013; European Commission 2017). Rules to determine reasonable royalties for patent

[4] In the supplementary appendix, we show that similar results can be obtained if firms engage in Bertrand competition.

infringement have historically been established in US courtrooms. The most prominent case is *Georgia-Pacific v. United States Plywood* in 1970. The so-called Georgia-Pacific factors detailed 15 factors that still serve as an important reference for courts.[5] However, this does not provide a precise definition of a reasonable royalty (Layne-Farrar et al., 2007; Geradin, 2013; Ménière et al., 2015). Subsequently, various simple rules have been proposed. One notable example is the *numeric proportionality rule* that distributes royalties according to the number patents essential to the standard. This has been proposed in several cases against Qualcomm in the EU and has also been used in patent pools (Layne-Farrar and Lerner, 2011). Although it reduces transaction costs, it seems neither fair nor reasonable.

When it comes to interpretation of FRAND, many economists and legal experts agree that a *reasonable* royalty should be based on hypothetical arms-length negotiation at the time the standard is being set (e.g. Swanson and Baumol 2005; Geradin 2013). This also follows from the patent law exemplified by Georgia-Pacific factors. For *non-discrimination*, a narrow definition requires the same royalty for all licensees, while a broader definition requires only that similar users should pay similarly (e.g.,Gilbert 2011; Carlton and Shampine 2013). Moreover, the principle should also extend to the owner herself (Swanson and Baumol, 2005). For *fairness*, there is hardly any paper discussing how to define it precisely in the context of FRAND.[6] As mentioned, the literature often uses FRAND and RAND interchangeably.

So how are FRAND terms determined? Swanson and Baumol (2005) suggest that non-discriminatory compensation should be determined by *Efficient Component Pricing Rule* (ECPR). As a pricing rule for service in public utility bottlenecks, it requires a vertically integrated patent holder to set the royalty to the price of the final good sold by the patent holder net of its marginal cost.[7] Extending to a standard with two complementary components, the ECPR implies that the sum of the royalties for two components cannot exceed the incremental value of the standard and the split of royalty revenue is decided by hav-

[5]For example, one factor requires the royalty to be an outcome from a hypothetical arm's length negotiation at the time of infringement, and one factor considers the opinion testimony of qualified experts.

[6]The problem of fair division has a long history. See e.g., Moulin (2004) and Hougaard (2009).

[7]They argue that ECPR is reasonable when there is a substitutable technology or downstream entry barrier is low. In particular, when two technologies are perfect substitutes, their ECPR-determined licensee fee implies zero compensation, which is consistent with our result.

ing the SSO to hold simultaneous auctions for each component (Layne-Farrar et al., 2007; Schmalensee, 2009). However, except for some special cases, there will be multiple equilibria, and it is not easy to determine which to select.

Efficiency-based rules such as ECPR take market outcomes as the benchmark, and do not explicitly consider equity. Various cooperative game theory concepts, such as the Shapley value, (Layne-Farrar et al., 2007; Dehez and Poukens, 2013; Dewatripont and Legros, 2013; Pentheroudakis and Baron, 2017) capture the fairness notion, but may offer payment to viable technologies that are unlikely to be included in the standard. In particular, Sidak (2013) criticizes the direct application of the Shapley value. For instance, a technology with a superior substitute should receive no payment under reasonableness. In our model, if the patent has zero incremental value to the standard, the holder of this patent does not receive compensation.

Regarding to market outcomes, Gilbert (2011) considers a Nash bargaining solution for FRAND licensing terms, and Lemley and Shapiro (2013) propose final arbitration by experts after bargaining breakdowns as a market-based implementation. Layne-Farrar and Llobet (2014) argue that ECPR may lead to an inefficient technology when the technology can be applied to multiple markets. Lerner and Tirole (2015) suggest that market outcome under price cap commitment is sufficient to restore ex-ante competition and efficiency, and thus there is no need to impose FRAND commitment.

Our paper sheds light on the recent growing literature on litigation issues related to FRAND compensation (e.g., Ratliff and Rubinfeld 2013; Langus et al. 2013; Sidak 2015; Choi 2016). These papers study how the royalty is determined in the bargaining under the shadow of FRAND court rulings. Without a universally accepted definition of FRAND, both the patent holder and implementers can exploit legal uncertainty, leading to opportunistic behavior. We suggest a precise definition of FRAND that services to resolve the dispute over different compensation rules and thereby reduces legal uncertainty.

2 Model

Let N denote an *industry* comprising a finite set of $n \geq 2$ individual firms: $N = \{1, \ldots, n\}$. Each firm $i \in N$ is endowed with a technology r_i. Let $R = \{r_1, ..., r_n\}$ denote the profile of technologies. We may think of the technology r_i as a package of all the patents owned, or controlled, by firm i. The SSO, representing the industry, selects a subset of technologies $S \subseteq R$ to form a standard. Hence, technology r_i is *standard-essential* if and only if $r_i \in S$. Therefore, N may consist of different types of firms: some firms may have a double role as standard-essential patent (SEP) owner (licenser) and implementer (licensee), while other firms may be pure implementers if their technologies are non-essential to the standard.

For each individual firm $i \in N$, we assume that the profit of firm i can be represented by a function $u_i : 2^R \to \mathbb{R}$, in a situation where i has free access to subsets of technologies $D \subseteq R$ when all other firms in N also have free access to these technologies and the firms compete under given market circumstances.[8] For instance, $u_i(R)$ is the profit of firm i when all firms have free access to the full body of knowledge provided by all technologies in R. Note that the function only specifies profits in the case where all firms share access to a common set of technologies. We normalize $u_i(\emptyset) = 0$ for all $i \in N$.

Now, let $w(D) = \sum_{i \in N} u_i(D)$ be the total (industry) profit when technology set $D \subseteq R$ is shared. We assume that a standard is always selected to maximize the total industry profit given the available technology set. In particular, this means that the standard $S \subseteq R$ is selected as a solution to the problem $\arg\max_{D \subseteq R} w(D)$. Note that several standards may result in the same maximum industry profit, in which case a given standard is selected at random among the set of optimal solutions.

For all $D \subseteq R$, let $v(D) = \max_{T \subseteq D} w(T)$ be the monotone cover of w. Thus, given the selected standard S, when technology set R is available, we have that $v(R) = v(S)$ and $v(S \cup \{r_i\}) - v(S) = 0$ for all non-standard-essential technologies $r_i \in R \setminus S$.

[8] Lerner and Tirole (2015) consider a model of SSO where firm heterogeneity is represented by parametric distribution θ that represents opportunity cost such that the value for firm $i \in N$ as $u_i(D) = u(D) - \theta$ for $D \subseteq R$ for some function u.

The problem: Given a standard S, we ask what is a "Reasonable" compensation to each firm j for giving members of the industry N access to its standard-essential technology $r_j \in S$ and, subsequently, how this compensation should be divided in a "Fair" way among members of N such that the result is "Non-Discriminatory."

We shall base our definition of a reasonable compensation on the monotone cover of the total industry profit function, v, while our definition of fair royalty payments will refer to the firm-specific profit functions u_i.

One practical issue is that firm-specific profits are private information, which the SSOs do not have direct access to. However, in Section 5 we illustrate possible ways to estimate them using market structure and demand in certain simple settings. Moreover, our framework can also encompass simpler versions to express firm values, as discussed in Section 4. In this sense our model can be adjusted to fit practice more closely on an ad-hoc basis.

3 Reasonable Compensation

Given the set R of available technologies, we submit that a *reasonable*[9] compensation to firm j should be proportional to the incremental profit of the industry N from having access to technology $r_j \in R$ when selecting the standard, i.e., the value,

$$M_j = v(R) - v(R \setminus \{r_j\}). \tag{1}$$

The incremental value M_j equals the upper bound of what the industry is willing to compensate its member, firm j, for adding its technology, r_j, to the pool of available technologies from which the standard can be selected. So if technology r_j is non-essential then $M_j = 0$, and non-essential technology owners receive no compensation. Moreover, if two technologies are perfect substitutes, the incremental value to the industry will be zero for both technologies and they will not be eligible for compensation. In an intermediate case where one

[9]Note that our usage of the term "reasonableness" follows the (F)RAND literature, which is different from the generic legal usage that represents "appropriateness" or "ordinary".

technology is a superior substitute to another technology, one should not expect the market to compensate the inferior one. Basically, this leaves (1) as the only relevant market-based solution in line with the Law and Economics literature that interprets the upper bound of a "reasonable" compensation as the incremental value over the next best alternative available in an ex-ante market (see e.g., Lemley and Shapiro 2013). It is also in line with court rulings that consider the upper bound to be the result of a hypothetical negotiation before setting the standard: in bargaining between two sellers with substitutable items, price is set competitively (e.g., *Microsoft v. Motorola*).

Note that defining a reasonable compensation according to (1) is also non-discriminatory since it is anonymous. The size of compensation does not depend on the labeling of firms. If two firms have the same incremental values, their compensations are identical.

A reasonable compensation scheme should avoid overcompensation (e.g. European Commission 2017), i.e., the total willingness to pay for access to the standard should not exceed the value to the industry of having access to the standard: $\sum_{r_j \in R} M_j \leq v(S)$. Otherwise, in case of *royalty stacking*, every patent holder expects to obtain the ex-ante incremental value of their technology, but the sum of these exceeds the market value of the end product. Thus, the compensation has to be lowered in order for the standard to be economically viable.[10] Here we suggest using a simple proportional down-scaling in line with our general fairness idea, i.e., as σM_j, for all $r_j \in R$, where $\sigma = v(S)/\sum_{r_j \in R} M_j$.

4 Fair and Non-Discriminatory Royalty Payment

We now turn to the question of how firms, as characterized by their individual profit functions u_i, should fairly compensate technology owner j for adding technology r_j to the common

[10] Shapiro (2001) argues that royalty stacking is a natural consequence of the dispersed ownership of technologies, similar to double marginalization. Siebrasse and Cotter (2016) and Pentheroudakis and Baron (2017) emphasize that a FRAND royalty should minimize the risk of royalty stacking. Recent court rulings (for example, the case *Microsoft* v. *Motorla* in 2012) also suggest the ruling should take royalty stacking into account.

pool.[11]

Fix the set of firms N and technologies R. A *royalty payment problem* is a pair (u, M) where u is the profile of firm-specific profit functions and $M = (M_1, \ldots, M_n)$ is the profile of technology compensations as derived in Section 3 above. Without loss of generality, we assume $\sum_{r_j \in R} M_j \leq v(R)$.

A *payment rule* assigns a vector of payments, $t(u, M) \in \mathbf{R}_+^n$, to any problem (u, M). For every firm i, t_i is the total royalty that firm i must pay to compensate technology owners (including themselves). We assume payment rules are budget-balanced in the sense that the total royalty payment adds up to the total compensation, i.e., $\sum_{i \in N} t_i = \sum_{r_j \in R} M_j$.

The challenge now is to define what we mean by a fair and non-discriminatory allocation rule, which will distribute the payment of the reasonable compensations, M, among the firms in N. In line with the traditional welfare theoretic approach, we will capture this by defining a set of requirements (axioms), each representing some normative aspect of royalty payments build upon FRAND terms.

Our first requirement is a classic separability property. We assume that any relevant payment rule is *Additive* in compensation. Formally, a payment rule t is additive in compensation if, for any compensation profiles, M and M', such that,

$$t(u, M + M') = t(u, M) + t(u, M'). \tag{2}$$

The additivity assumption is standard in the literature on fair allocation (Moulin, 2004) and it implies that payment rules take the form

$$t(u, M) = \sum_{r_j \in R} y(u, r_j) M_j, \tag{3}$$

where $y(u, r_j) \in \Delta(N)$ (with $\Delta(N)$ being the $N-$simplex) specifies how the compensation

[11] We have not specified whether the royalty is per unit or lump-sum. Our framework can incorporate both cases. To calculate lump-sum licensing fees, the social and private values are based on incremental total profits due to the standard. To calculate the per-unit licensing fees, the social and private values are based on the increment per unit profit due to the standard.

M_j for technology r_j is shared relatively among firms in N (Hougaard and Moulin, 2014). We will talk about $y(u, r_j) = (y_1(u, r_j), \ldots, y_n(u, r_j))$ as a profile of firm-specific *liabilities* for compensation of technology $r_j \in R$: that is, the royalty payment of firm i to technology owner j is given by $y_i(u, r_j)M_j$.

By focusing on additive payment rules, we emphasize that fair liabilities (and thereby royalty payments) for a given technology r_j do not depend on the size of compensation M_j, or the size of any other technology compensation for that matter, but rather on the firm-specific profit functions: the way that the individual firms benefit from using the available technologies of the pool. This seems to play a crucial role in incentivizing innovation since it ensures that liabilities are correlated with individual firm's value, given the market structure.

We will further assume that any relevant payment rule satisfies *Anonymity*, i.e., that ensures payments are independent of the labeling of the firms. Indeed, this is a basic requirement of non-discrimination (Moulin, 2004). Formally, a payment rule is anonymous if, for any permutation π of N, such that,

$$t(\pi(u), M) = \pi(t(u, M)). \tag{4}$$

Note that this implies that a patent holder has to pay the same rate as other implementers. In specific contexts, we may want to impose stronger versions of non-discrimination. We can easily incorporate them into our model, which will be elaborated in Section 5. To define "non-discrimination" as a form of anonymity among participating firms may (and should) lead to price discrimination because firms perceive the value of various patents differently, which ought to be reflected in the royalties. However, this viewpoint differs from that of many practitioners who may see non-discrimination primarily as an obligation to license direct competitors in order to avoid foreclosure in downstream markets (e.g., see discussion in Carlton and Shampine 2013).

Moreover, we will require that the rule is *Consistent*[12]: remaining firms' royalty payments

[12] Note that, technically speaking, consistency requires that we work with a variable populations framework which we avoid here for simplicity of notation since we do not aim at presenting a formal axiomatic

are unchanged, if we are to remove one firm from the industry after it has paid its royalty. Formally, the consistency assumption under additive payment rules implies that, for any problem (u, M) and technology $r_j \in R$, the profile of firm-specific liabilities when firm i departs after paying and receiving its royalty is

$$y_{-i}(u, r_j) = (1 - y_i(u, r_j)) \times y(u^{-i}, r_j) \tag{5}$$

where (u^{-i}, r_j) is the reduced problem where firm i is excluded from N and u^{-i} is the profile of the remaining firms' profit functions.

As shown in Hougaard and Moulin (2014), anonymity together with consistency (given additivity) implies that liabilities $y(u, r_j)$ are proportional to a given *liability index* $\ell(u_i, r_j)$ specific to each firm i, i.e.,

$$y_i(u, r_j) = \frac{\ell(u_i, r_j)}{\sum_{h \in N} \ell(u_h, r_j)} \quad \text{for all } i \in N \tag{6}$$

where $\ell(u_i, r_j) \geq 0$ is firm i's liability index for technology r_j. So combining with (3) we get that total royalty payments take the form

$$t_i(u, M) = \sum_{r_j \in R} \frac{\ell(u_i, r_j)}{\sum_{h \in N} \ell(u_h, r_j)} M_j, \quad \text{for all firms } i \in N.\text{[13]} \tag{7}$$

As mentioned above the individual properties — additivity, anonymity and consistency — are all well-established and normatively compelling requirements from the theory of fair allocation (see e.g., Thomson 2012 for further justification). The consequence of applying them together, i.e., fairness in the form of proportionality to some individual characteristic, can be traced all the way back to Aristotle's writings on distributional justice.

Yet, we must still argue for a desirable liability index $\ell(\cdot, \cdot)$. For each firm $i \in N$ the pair (R, u_i) constitutes a cooperative game (where the technologies can be construed as the "players"). Therefore, one obvious suggestion, in line with the conventional approach in the

characterization of our suggested indices.

[13] For the sake of completeness we provide the formal statement and its proof in the Appendix.

cost-sharing literature, would be to use solution concepts from the theory of cooperative games as liability indices: for instance, the celebrated Shapley value,

$$\ell^S(u_i, r_j) = s_j(R, u_i) = \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n - |D| - 1)!}{n!}(u_i(D \cup r_j) - u_i(D)), \tag{8}$$

for all $r_j \in R$. The normative foundation of the Shapley value is well known and there exists several axiomatic characterizations, see e.g., Shapley (1953); Peleg and Sudhölter (2007), which in principle can be combined with the three requirements of additivity, anonymity, and consistency in order to produce an axiomatic foundation of compensation in the form of (7) when using the Shapley liability index (8).

Example 1 *Consider, as in Layne-Farrar et al. (2007), three technologies where technology 1 is necessary and technologies 2 and 3 are imperfect substitutes. In particular, we have* $v(\{r_1, r_2\}) = v(\{r_1, r_2, r_3\}) = 1 + \delta > 1 = v(\{r_1, r_3\})$ *and* $v(D) = 0$ *otherwise. Computing the Shapley value of the game* (N, v)*, Layne-Farrar et al. (2007) find compensations to firms 1, 2, and 3 are* $M_1^s = \frac{2}{3} + \frac{\delta}{2}$*,* $M_2^s = \frac{1}{6} + \frac{\delta}{2}$*, and* $M_3^s = \frac{1}{6}$*, respectively.*

Using our approach, reasonable compensations should be determined by (1), i.e., as the marginal contributions of technologies 1, 2, and 3, respectively, to the industry: $M_1 = 1 + \delta, M_2 = \delta$*, and* $M_3 = 0$*; with gross compensation being* $(1 + \delta)\sigma$*,* $\delta\sigma$*, and* 0 *where* $\sigma = (1 + \delta)/(1 + 2\delta)$*. Clearly, this differs from the above Shapley compensations:* $\sigma M_1 \geq M_1^s$ *for all* $\delta \geq 0$*, while* $\sigma M_2 \leq M_2^s$ *for* $\delta \in [0, 1]$*; thus, our approach gives more compensation to firm 1 and less to firm 3, while for firm 2, it depends on the size of* δ*. The Shapley compensation gives a positive compensation to firm 3 (being 1/6), but this is unfortunate since it may lead to patent thicket as noted in Shapiro (2001). In contrast, our approach coincides with the market/efficiency-based approach by Swanson and Baumol (2005) because competition between firms 2 and 3 will drive the compensation of firm 3 to zero.*

Moreover, our approach also determines how this (reasonable) compensation is shared among the firms in the form of royalty payment. In particular, firm-specific profits can be given by $u_1(\{r_1, r_3\}) = \frac{3}{4}$*,* $u_2(\{r_1, r_3\}) = u_3(\{r_1, r_3\}) = \frac{1}{8}, u_1(D) = \frac{3}{4}(1 + \delta); u_2(D) =$

$u_3(D) = \frac{1}{8}(1+\delta)$ for $D = R, \{r_1, r_2\}$, and $u_i(D) = 0$ otherwise.[14] *The Shapley liability index (8) for firm 1, with respect to technology r_1, is $s_1(R, u_1) = \frac{1}{3}(u_1(R) - u_1(\{r_2, r_3\})) + \frac{1}{3}u_1(\{r_1\}) + \frac{1}{6}(u_1(\{r_1, r_2\}) - u_1(\{r_2\})) + \frac{1}{6}(u_1(\{r_1, r_3\}) - u_1(\{r_3\})) = \frac{1}{8}(4+3\delta)$. Since the sum of liabilities of all firms for r_1 is $\frac{1}{6}(4+3\delta)$, firm 1's proportional liability is $y_1(u, r_1) = \frac{3}{4}$. Similarly, we can show that $y_1(u, r_2) = y_1(u, r_3) = \frac{3}{4}$. For firms 2 and 3, liabilities are identical for all technologies and we get $y_2(u, r) = y_3(u, r) = \frac{1}{8}$ for all $r \in R$.*

Hence, the total royalty paid by firm 1 is $\frac{3}{4}\sigma\delta$ to firm 2, and 0 to firm 3, while firm 1 receives $\frac{2}{8}(1+\delta)\sigma$ in total from firms 2 and 3. Similarly, firm 2 pays $\frac{1}{8}(1+\delta)\sigma$ to firm 1 and 0 to firm 3, while firm 2 receives $(\frac{3}{4} + \frac{1}{8})\delta\sigma$ in total from firms 1 and 3. Therefore, firm 3 pays $\frac{1}{8}(1+\delta)\sigma$ to firm 1 and $\frac{1}{8}\delta\sigma$ to firm 2, while receiving no payment from the other firms.

Finally, the payoff for firm 1 is $\frac{3}{4}(1+\delta) - \frac{3}{4}\sigma\delta + \frac{2}{8}(1+\delta)\sigma = \frac{(1+\delta)^2}{1+2\delta}$. Similarly, the payoff for firm 2 is $\frac{1}{8}(1+\delta) - \frac{1}{8}(1+\delta)\sigma + (\frac{3}{4} + \frac{1}{8})\sigma\delta = \frac{\delta(1+\delta)}{1+2\delta}$ while the payoff for firm 3 is $\frac{1}{8}(1+\delta) - \frac{1}{8}(1+\delta)\sigma - \frac{1}{8}\delta\sigma = 0$. □

Besides the Shapley value, we would like to emphasize the fact that for individual profit functions inducing a game in technologies (R, u_i), any compelling solution concept from cooperative game theory can be applied as a potential liability index.

The overall approach can also work with more crude representations of firms' preferences. For instance, Hougaard and Moulin (2014) analyze a model where agents' preferences are dichotomous: either a given subset of technologies serves the needs of the firm or not. Minimal serving sets are those subsets of technologies for which removing any single technology renders the firm unserved. If firms differ in their structure of minimal serving sets there are good reasons for their laibility index to differ as well. Hougaard and Moulin (2014) introduce and characterize the so-called counting liability index for a given technology, defined as the ratio between the number of (minimal) serving sets containing this given technology and the total number of (minimal) serving sets of the firm. Clearly, the type of information required

[14] These profit functions can be rationalized using a standard Cournot setting, with one upstream firm, two downstream firms, and linear demand.

to establish the minimal serving sets is much less demanding than estimating firms' profit functions. For SEPs, this type of "counting" liability index will typically render the firms symmetric and therefore result in equal licensing fees. In particular, when there are many patents such that the values of each technology are similar, it further boils down to "patent counting" or "numeric proportionality," such that licensing fees should be proportional to the number of patents, as often adopted in patent pools (Layne-Farrar and Lerner, 2011).

5 Application

We now illustrate our approach by considering two different market structures: (1) horizontal — all firms are downstream producers and (2) vertical — some firms are upstream producers, and some firms are downstream producers.[15] Following the literature in industrial organization, we focus on cost-reducing innovations, which is isomorphic to value enhancing innovation (Tirole, 1988).

In the following, when we determine liabilities of individual firms, we mean liabilities in the form of (6) using the Shapley liability index (8), unless explicitly stated otherwise. To simplify exposition, we assume every technology is included in the standard, i.e. $S = R$.

5.1 Horizontal Market

Following the innovation literature initiated by Arrow (1962), we consider process innovations in a homogeneous goods market. A prominent example is portable storage devices such as memory card/sticks where the homogeneous goods assumption is a good approximation.

Firms are symmetric in the sense that they have identical cost functions — except for fixed cost —and face the same market demand. This implies that in every symmetric equilibrium, for every firm $i \in N$, we have $u_i(D) = \bar{u}(D) - \theta_i$ for all $D \subseteq R$, where θ_i is the fixed cost of firm i and $\bar{u}(D)$ is the equilibrium profit when all firms have access to D.[16] The

[15] We focus on Cournot competition for the downstream market, noting that Bertrand competition would deliver similar results.

[16] This is equivalent to Lerner and Tirole (2015) where they consider firms that have different opportunity costs for using a technology.

compensation for individual technologies may differ, but the liabilities of each firm, for each technology, are identical across firms. We record this observation in Proposition 1, below.

Proposition 1 *Consider a horizontal cooperative agreement between n symmetric firms. Under FRAND compensation, firm i's liability for technology $r_j \in R$ is, $y_i(u, r_j) = \frac{1}{n}$.*

To illustrate the result, we consider a Cournot model with linear demand, zero fixed cost, and constant marginal cost. When firms have access to the set of technologies $D \subseteq R$, they have a constant marginal cost c_D.

Example 2 *The inverse market demand is given by $P = a - Q$ where $Q = \sum_{i \in N} q_i$ is the aggregate production and q_i is production by firm $i \in N$. The profit of firm i with access to technology D is $u_i(D) = (p - c_D)q_i$. Standard calculation shows that the equilibrium production and profit of firm i are $q_i(D) = \frac{a-c_D}{n+1}$ and $\pi_i(D) = q_i^2(D)$. Therefore, when each firm has access to R, firm i pays total royalty $t_i = \sum_{r_j \in R} y_i(u, r_j)M_j$.*[17] *Thus, firm i's per unit royalty for technology j becomes $\tau_i(j) = \frac{y_i(u,r_j)M_j}{q_i(R)} = \frac{M_j}{nq_i(R)}$.*

It is useful to compare the result of our model to some existing rules. Under the symmetric case, our approach should be similar to some of the rules adopted in the literature, since non-discrimination implies fairness under a symmetric setup. Sidak (2013) *mentions two methodologies for calculating FRAND royalty: (i) top-down and (ii) proportional contribution. We will show that our rule leads to the same outcome as the top-down approach and yields similar outcomes to the proportional contribution approach.*

(i) The top-down approach considers per unit FRAND royalty as: aggregate royalty burden $\times$ $\frac{\textit{Contribution of Patent}}{\textit{Value of standard}}$. If the standard is essential to the product, the aggregate royalty burden is the profit margin. Assuming that the standard involves all the technologies, and the value of standard is $v(R)$,[18] *we have per unit royalty $= (p - c_R) \times \frac{M_j}{v(R)} = (p - c_R) \times \frac{M_j}{\sum_{i \in N}(p-c_R)q_i(R)} = \frac{M_j}{nq_i(R)} = \tau_i(j)$. Thus, our approach delivers the exact same result.*

[17]Note that the compensation to technology r_j would be σM_j if $\sum_{r_j \in R} M_j > v(R)$.

[18]Strictly speaking, the value of the standard should be $v(R) - v(\emptyset)$. We have normalized $v(\emptyset) = 0$, which holds when the market would not exist without any of technology $r \in R$. In the current Cournot case, the condition is equivalent to $c_\emptyset \geq a$. Hence, if $v(\emptyset) > 0$, our FRAND royalty is different from the top-down approach.

(ii) proportional contribution considers per unit FRAND royalty as: Price $\times \frac{\textit{Contribution of Standard}}{\textit{Value of Product}} \times \frac{\textit{Contribution of Patent}}{\textit{Value of standard}}$. *Since the standard is fully utilized by all firms, we assume that the "contribution of standard to value of product" is 1. We then have* $p \times \frac{M_j}{v(R)} = p \times \frac{M_j}{(p-c_R)\sum_{i\in N} q_i(R)} = \frac{p}{p-c_R} \times \frac{M_j}{nq_i(R)} = \frac{p}{p-c_R} \times \tau_i(j)$. *That is, our approach delivers similar results to the proportional contributions rule adjusted by the relative markup.*□

Proposition 1 shows that when firms are symmetric, the resulting FRAND liabilities are the same. It is natural to expect that when firms are ex-ante asymmetric, it may lead to differences in firm liabilities. To illustrate this, we consider the linear Cournot case as in Example 2, but the marginal cost of production for every firm $i \in N$ with technology $D \subseteq R$ is $c_i(D)$. Let $q_i(D)$ and $\pi_i(D)$ denote the equilibrium quantity and profit of firm i when all firms have access to D. Hence, the private value for firm i is $u_i(D) = \pi(D) - \pi(\emptyset)$.

Example 3 *Focusing on the two-firm case. Under FRAND compensation firm 1's liability for technology* $r_1 \in R$, *is* $y_1(u, r_1) = \left(1 + \frac{q_2^2(R)-q_2^2(\{r_2\})+q_2^2(\{r_1\})-q_2^2(\emptyset)}{q_1^2(R)-q_1^2(\{r_2\})+q_1^2(\{r_1\})-q_1^2(\emptyset)}\right)^{-1}$. *Hence, firm 1's per unit FRAND royalty for technology* r_1 *is* $\frac{y_1(u,r_1)M_1}{q_1(R)} = \frac{M_1}{q_1(R)\left(1+\frac{q_2^2(R)-q_2^2(\{r_2\})+q_2^2(\{r_1\})-q_2^2(\emptyset)}{q_1^2(R)-q_1^2(\{r_2\})+q_1^2(\{r_1\})-q_1^2(\emptyset)}\right)}$.

For comparison, the top-down approach yields the per unit royalty as $(p - c_1(R)) \frac{M_1}{v(R)-v(\emptyset)} = \frac{M_1}{q_1(R)\left(1+\frac{q_2^2(R)-q_2^2(\emptyset)-q_1^2(\emptyset)}{q_1^2(R)}\right)}$. *So according to the top-down approach, royalty payment depends on the ratio of total profit. The proportional contribution approach gives the per unit royalty as* $p \times \frac{M_1}{v(R)-v(\emptyset)} = \frac{p}{p-c_1(R)} \times \frac{M_1}{q_1(R)\left(1+\frac{q_2^2(R)-q_2^2(\emptyset)-q_1^2(\emptyset)}{q_1^2(R)}\right)}$. *Both the top-down and proportional contribution approaches are different from our approach as they only rely on the case where firms have access to all technologies, but not any other possible cases where only subsets of technologies are available. Moreover, our approach focuses on cost-reducing technologies and thereby on marginal changes of the profit due to technology adaption. Hence, when all technologies are truly essential (i.e., all have to be present to produce value:* $q_i(\emptyset) = q_i(\{r_1\}) = q_i(\{r_2\}) = 0$ *for all* $i \in N$*), our approach coincides with the top-down rule.*□

The following proposition further illustrates our approach in the standard case that the marginal cost of production is $c_i(D) = c_i(\emptyset) - \theta_i\epsilon_D$, where $c_i(\emptyset)$ is the marginal cost of firm i with existing technology (not using any technology in R) and ϵ_D is the cost saving brought by technology $D \subseteq R$. For simplicity, suppose there are two firms and technologies that are perfectly compatible ($\epsilon_R = \epsilon_{r_1} + \epsilon_{r_2}$).

Proposition 2 *Consider a horizontal cooperative agreement between two firms engaging in Cournot competition and producing homogeneous products. Suppose the technologies are perfectly compatible ($\epsilon_R = \epsilon_{r_1} + \epsilon_{r_2}$). Under FRAND compensation, (a) the liability for $r_j \in \{r_1, r_2\}$ by firm $i \in \{1,2\}$ is $y_i(u, r_j) = \left(1 + \frac{(2\theta_k - \theta_i)\sum_{D\subseteq R} q_k(D)}{(2\theta_i - \theta_k)\sum_{D\subseteq R} q_i(D)}\right)^{-1}$ where $k = \{1,2\}\backslash\{i\}$. (b) In particular, (i) when both firms have the same efficiency parameter ($\theta_1 = \theta_2$), we have $y_i(u, r_j) = \left(1 + \frac{\sum_{D\subseteq R} q_k(D)}{\sum_{D\subseteq R} q_i(D)}\right)^{-1}$ where $k = \{1,2\}\backslash\{i\}$, and (ii) when both firms have same marginal cost under the same access to technologies ($c_1(D) = c_2(D)$ for all $D \subseteq R$), we have $y_i(u, r_j) = \frac{1}{2}$.*

As such, payment depends on the efficiency parameters (θ_i): the more efficient firm of the two will pay more in royalty than the less efficient firm, simply because access to the cost-reducing technology is more valuable to the more efficient firm. This is not in conflict with non-discrimination as similar firms are treated similarly.

5.2 Vertical Market

We consider the case where upstream firms produce intermediate goods or provide resources to downstream firms under a vertical cooperative agreement, see e.g., Kim (2004); Dewatripont and Legros (2013).

Suppose there are m upstream and l downstream firms. Let M and L be the sets of upstream and domstream firms, respectively. Each downstream firm produces one unit of the final product using one unit of intermediate input from each upstream firm. An upstream firm i charges d_i for intermediary inputs that are necessary for downstream production. When firms have access to the set of technologies $D \subseteq R$, they have a constant marginal cost c_D. The profit of downstream firms $i \in L$ with access to technologies D is $\pi_i = (\alpha - \beta Q - \sum_{k\in M} d_k - c_D)q_i$ and the profit for upstream firms $k \in M$ are $\pi_k = d_k Q$ where $Q = \sum_{i\in L} q_i$.

Consider a two-stage game where all upstream firms first engage in Bertrand competition, and subsequently all downstream firms engage in Cournot competition.

Proposition 3 *The liability for technology $r_j \in R$ by an upstream firm $i \in M$ is $y_i(u, r_j) = \frac{l+1}{m(l+1)+1}$, and the liability for downstream firm $k \in L$ is $y_k(u, r_j) = \frac{1}{l(m(l+1)+1)}$.*

Note that the liabilities depend on the market structure, but not on the market characteristics given by parameters α and β. As the number of upstream firms increases, liabilities for all firms reduce. On the other hand, as the number of downstream firms increases, the liabilities for upstream firms increase, while those for downstream firms decrease.

6 Concluding Remarks

Inspired by the literature on fair allocation, we adapt an axiomatic approach to FRAND royalties. Our approach has several advantages. The characterizing properties of our suggested payment rule are normatively compelling axioms that are widely accepted in the literature on fair allocation (Hougaard and Moulin, 2014). The definition of a reasonable compensation satisfies "no contribution, no pay," so inferior substitute technologies have no value to the industry and therefore are not eligible for payment (Sidak, 2013). This is in line with the market-based idea of Swanson and Baumol (2005). It further accounts for royalty stacking as well as patent hold-up, and thereby preserves the economic viability of the standard (Pentheroudakis and Baron, 2017), and delivers clearly specified royalty payments, including what the patent owners should pay themselves (a question posed by Swanson and Baumol 2005).

Moreover, our rule takes the market structure into account via firm-specific profit functions, which ensures that the royalty payment is in line with the market value of individual firms for having access to the technology pool. It is also in line with conventional royalty rules like *top-down* and *proportional-contribution* rules in the simple case of completely symmetric firms (here fairness coincides with non-discrimination). In the more complicated case of asymmetric firms, our royalty payment explicitly depend on market conditions: our approach coincides with the top-down rule only in cases where all technologies are truly essential. This reflects the flexibility in the FRAND terms: there is no obligation to ensure that every licensee is receiving identical terms (Epstein et al., 2012).

Our modeling framework is also useful for a planner/regulator who is interested in finding fair and reasonable compensations to promote industry-wide cooperation. Examples include patent pools (Lerner and Tirole, 2004), research joint ventures (Katz, 1986), and platform markets (Church and Gandal, 1992; Rochet and Tirole, 2003).

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A Appendix: Omitted Statements and Proofs

Formal argument for Eq. 7

Here we provide a formal argument for eq. (7) in the text.

Our sketch of proof largely follows (Hougaard and Moulin, 2014). It is easy to check the rule in eq. (7) satisfies additivity, anonymity, and consistency. Consequently, we focus on showing the sufficiency. It is well-known that (2) implies (3). We therefore show that (4) and (5) imply (6). Fix the set of technologies R and a given $r_j \in R$. By (4) there exists, for all n, a function $\xi^n(u; u_1, \ldots, u_{n-1})$ symmetric in the $n-1$ variables $u_1, \ldots, n_{n-1}$ and such that $y_i(u, r_j) = \xi^n(u_i; \{u_z\}_{z \in N \setminus \{i\}})$ for all profiles of u.

Assuming that r_j is such that all $\xi^n(u; u_1, \ldots, u_{n-1}) > 0$ we get for $N = \{1, 2, 3\}$ and applying (5) that

$$\frac{\xi^3(u_1; u_2, u_3)}{\xi^3(u_2; u_1, u_3)} = \frac{\xi^2(u_1; u_2)}{\xi^2(u_2; u_1)} = \rho(u_1, u_2).$$

Permuting the role of the agents we get $\rho(u_1, u_2) \times \rho(u_2, u_3) \times \rho(u_3, u_1) = 1$. Therefore $\rho(u_1, u_2) = \frac{f(u_1)}{g(u_2)}$ for some positive functions f and g. Since $\rho(u_1, u_2) \times \rho(u_2, u_1) = 1$ we take $f = g = \ell$ which implies that

$$\frac{\xi^2(u_1; u_2)}{\ell(u_1} = \frac{\xi^2(u_2; u_1)}{\ell(u_2)} \quad \Rightarrow \quad \frac{\xi^3(u_1; u_2, u_3)}{\ell(u_1)} = \frac{\xi^3(u_2; u_1, u_3)}{\ell(u_2)}.$$

Repeated applications of (5) give

$$\frac{\xi^n(u_1; \{u_z\}_{z \in N \setminus \{1\}})}{\ell(u_1)} = \frac{\xi^n(u_2; \{u_z\}_{z \in N \setminus \{2\}})}{\ell(u_2)},$$

for all n. Therefore y takes the form (6). □

Proof of Proposition 1

When firms are symmetric $u_i(D) = \bar{u}(D)$ for all $D \subseteq R$ and $i \in N$. Indeed, since all firms have access to the same (sub)set of technologies they face the same production costs. As an industry, they further face the same market demand. Thus, in equilibrium all firms have the same profit. Consequently, for each firm the induced game $(R, u_i) = (R, \bar{u})$, making liabilities of the form (6), equal to $1/n$, for each firm.

Proof of Proposition 2

Given the inverse market demand function for firm $i \in N$ is $p = \alpha - \beta \sum_{k \in N} q_k$, the profit maximization problem for firm i is given by

$$\max_{q_i} \pi_i(D) = (p_i - c_i(D))\, q_i = \left(\alpha - \beta \sum_{k \in N} q_k - c_i(D)\right) q_i,$$

where $k = 1, 2$ and $k \neq i$. The FOC for firm i is

$$\alpha - \beta \sum_{k \in N} q_k - \beta q_i(D) - c_i(D) = 0.$$

Solving all n FOCs, we have

$$q_i(D) = \frac{\alpha + \sum_{k \in N} c_k(\emptyset) - (n+1)c_i(\emptyset) + \left(n\theta_i - \sum_{k \in N \setminus \{i\}} \theta_k\right)\epsilon_D}{\beta(n+1)} \text{ and}$$

$$\pi_i(D) = \beta q_i^2(D) = \frac{1}{\beta}\left(\frac{\alpha + \sum_{k \in N} c_k(\emptyset) - (n+1)c_i(\emptyset) + \left(n\theta_i - \sum_{k \in N \setminus \{i\}} \theta_k\right)\epsilon_D}{n+1}\right)^2$$

The Shapley value for technology r_j is defined as

$$\begin{aligned} s_j(R, u_i) &= \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!\,(n - |D| - 1)!}{n!}\left(u_i(D \cup \{r_j\}) - u_i(D)\right) \\ &= \frac{n\theta_i - \sum_{j \neq i} \theta_j}{(n+1)^2} \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!\,(n - |D| - 1)!}{n!}\left(\epsilon_{D \cup \{r_j\}} - \epsilon_D\right)\left(q_i(D \cup \{r_j\}) + q_j(D)\right), \end{aligned}$$

so that the liability index is

$$\begin{aligned} & y_i(u, r_j) \\ &= \frac{s_j(R, u_i)}{\sum_{h \in N} s_j(R, u_h)} \\ &= \left(1 + \sum_{h \in N \setminus \{i\}} \frac{n\theta_h - \sum\limits_{k \neq h} \theta_k}{n\theta_i - \sum\limits_{k \neq i} \theta_k} \frac{\sum\limits_{D \subseteq R \setminus \{r_j\}} |D|!\,(n - |D| - 1)!\left(\epsilon_{D \cup \{r_j\}} - \epsilon_D\right)\left(q_h(D \cup \{r_j\}) + q_h(D)\right)}{\sum\limits_{D \subseteq R \setminus \{r_j\}} |D|!\,(n - |D| - 1)!\left(\epsilon_{D \cup \{r_j\}} - \epsilon_D\right)\left(q_i(D \cup \{r_j\}) + q_j(D)\right)}\right)^{-1}. \end{aligned}$$

Under perfect compatibility, we have

$$\begin{aligned} & y_i(u, r_j) \\ &= \left(1 + \sum_{h \in N \setminus \{i\}} \frac{n\theta_h - \sum_{k \neq h} \theta_k}{n\theta_i - \sum_{k \neq i} \theta_k} \frac{\sum_{D \subseteq R \setminus \{r_j\}} |D|!\,(n - |D| - 1)!\left(q_h(D \cup \{r_j\}) + q_h(D)\right)}{\sum_{D \subseteq R \setminus \{r_j\}} |D|!\,(n - |D| - 1)!\left(q_i(D \cup \{r_j\}) + q_i(D)\right)}\right)^{-1}. \end{aligned}$$

When $n = 2$, we have

$$y_i(u, r_j) = \left(1 + \frac{2\theta_k - \theta_i}{2\theta_i - \theta_k} \frac{\sum_{D \subseteq R} q_k(D)}{\sum_{D \subseteq R} q_i(D)}\right)^{-1} \text{ where } k = \{1, 2\} \setminus \{i\}.$$

When $\theta_1 = \theta_2$, then we have

$$y_i(u, r_j) = \left(1 + \frac{\sum_{D \subseteq R} q_k(D)}{\sum_{D \subseteq R} q_i(D)}\right)^{-1} \text{ where } k = \{1, 2\} \backslash \{i\}.$$

Furthermore if $c_i(D) = c_j(D)$ for all $D \subseteq R$, then $y_i(u, r_j) = \frac{1}{2}$.

Proof of Proposition 3

Given the inverse demand for downstream firms $i \in L$ is $p = \alpha - \beta \sum_{k \in L} q_k$, the profit maximization problem for firm i is given by

$$\max_{q_i} \pi_i(D) = (\alpha - \beta \sum_{h \in L} q_h - \sum_{h \in M} d_h - c_D) q_i.$$

The FOC for firm i is $\alpha - \beta \sum_{h \in L} q_h - \sum_{h \in M} d_h - c_D = \beta q_i$ so that solving l FOCs, we have

$$q_i(D) = \frac{\alpha - c_D - \sum_{h \in M} d_h}{\beta(l+1)} \text{ and } \pi_i(D) = \beta q_i^2(D) \text{ for all } i \in L.$$

By backward induction, the profit maximization problem for the upstream firm $k \in M$ is given by

$$\max_{d_k} \pi_k(D) = d_k \sum_{h \in L} q_h(D).$$

The FOC for firm $j \in M$ is $\alpha - c_D - \sum_{k \in M} d_k = d_j$ so that we have

$$d_j(D) = \frac{\alpha - c_D}{m+1} \text{ and } \pi_j(D) = \frac{l}{\beta(m+1)^2(l+1)}(\alpha - c_D)^2 \text{ for all } j \in M.$$

Therefore, we have

$$\pi_i(D) = \frac{1}{\beta(m+1)^2(l+1)^2}(\alpha - c_D)^2 \text{ for all } i \in L.$$

For downstream firms $i \in L$, the Shapley value for (R, u_i) for technology $r_j \in R$ is

$$\begin{aligned} s_j(R, u_i) &= \sum_{D \subseteq R \backslash \{r_j\}} \frac{|D|!(n - |D| - 1)!}{n!} [u_i(c_{D \cup \{r_j\}}) - u_i(c_D)] \\ &= \frac{1}{\beta(m+1)^2(l+1)^2} \sum_{D \subseteq R \backslash \{r_j\}} \frac{|D|!(n - |D| - 1)!}{n!} \left[(\alpha - c_{D \cup \{r_j\}})^2 - (\alpha - c_D)^2\right]. \end{aligned}$$

and for upstream firms $k \in M$, the Shapley value for (R, u_k) technology $r_j \in R$ is

$$\begin{aligned} s_j(R, u_k) &= \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n-|D|-1)!}{n!} [u_j(c_{D \cup \{r_j\}}) - u_j(c_D)] \\ &= \frac{l}{\beta(m+1)^2(l+1)} \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n-|D|-1)!}{n!} \left[(\alpha - c_{D \cup \{r_j\}})^2 - (\alpha - c_D)^2 \right]. \end{aligned}$$

Hence, for each technology $r_j \in R$, we have for each downstream firm $i \in L$

$$\begin{aligned} y_i(u, r_j) &= \frac{s_j(R, u_i)}{\sum_{h \in L} s_j(R, u_h) + \sum_{h \in M} s_j(R, u_h)} \\ &= \frac{\frac{1}{\beta(m+1)^2(l+1)^2}}{l \frac{1}{\beta(m+1)^2(l+1)^2} + m \frac{l}{\beta(m+1)^2(l+1)}} = \frac{1}{l\,(m\,(l+1)+1)}, \end{aligned}$$

and for each upstream firm $k \in M$

$$\begin{aligned} y_k(u, r_j) &= \frac{s_j(R, u_k)}{\sum_{h \in L} s_j(R, u_h) + \sum_{h \in M} s_j(R, u_h)} \\ &= \frac{\frac{l}{\beta(m+1)^2(l+1)}}{l \frac{1}{\beta(m+1)^2(l+1)^2} + m \frac{l}{\beta(m+1)^2(l+1)}} = \frac{l+1}{m(l+1)+1}. \end{aligned}$$

B Supplementary Appendix – Not For Publication

B.1 Multiple Product Market

Here we extend our discussion of single production market to multiple product markets in Section 5.

B.1.1 Horizontal Markets

Following Schmalensee (2009), we consider competing firms producing heterogeneous products. The inverse demand function by firm i is $p_i = \alpha_i - \beta_i q_i - \gamma \sum_{j \in N \setminus \{i\}} q_j$.[19] As assumed above, the marginal cost of production is $c_i(D) = c_i(\emptyset) - \theta_i \epsilon_D$. For exposition, we assume the market consists of two firms.

Proposition 4 *Consider a horizontal cooperative agreement between 2 firms engaging in Cournot competition, producing heterogeneous products. Suppose technologies are perfectly compatible ($\epsilon_R = \epsilon_{r_1} + \epsilon_{r_2}$). Under FRAND compensation, (a) the liability for $r_j \in \{r_1, r_2\}$ by firm $i \in \{1,2\}$ is*

$$y_i(u, r_j) = \frac{1}{1 + \frac{(\theta_k(2-\beta_i^2)-\gamma\theta_i)\sum_{D\subseteq R} q_k(D)}{(\theta_i(2-\beta_k^2)-\gamma\theta_k)\sum_{D\subseteq R} q_i(D)}} \text{ where } k = \{1,2\}\setminus\{i\}.$$

(b) In particular, (i) when both firms have the same productive efficiency ($\theta_1 = \theta_2$), we have

$$y_i(u, r_j) = \frac{1}{1 + \frac{(2-\beta_i^2-\gamma)\sum_{D\subseteq R} q_k(D)}{(2-\beta_k^2-\gamma)\sum_{D\subseteq R} q_i(D)}} \text{ where } k = \{1,2\}\setminus\{i\},$$

(ii) when both firms face symmetric markets ($\beta_1 = \beta_2 = \beta$), we have

$$y_i(u, r_j) = \frac{1}{1 + \frac{(\theta_k(2-\beta^2)-\gamma\theta_i)\sum_{D\subseteq R} q_k(D)}{(\theta_i(2-\beta^2)-\gamma\theta_k)\sum_{D\subseteq R} q_i(D)}} \text{ where } k = \{1,2\}\setminus\{i\}, \text{ and}$$

(iii) when firms are completely symmetric ($\theta_1 = \theta_2$ and $\beta_1 = \beta_2 = \beta$), we have

$$y_i(u, r_j) = \frac{1}{2}.$$

Proof. Given the inverse demand function for firm $i = 1, 2$ is $p_i = \alpha_i - \beta_i q_i - \gamma q_k$ where $k \neq i$, the profit maximization problem for firm i is given by

$$\max_{q_i} \pi_i(D) = (p_i - c_i(D))\, q_i = (\alpha_i - \beta_i q_i - \gamma q_k - c_i(D))\, q_i.$$

[19] Singh and Vives (1984) show that for the case of two firms that the demand function follows from the representative consumers that maximizes $U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i$ where $U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2)/2$ where α_i and β_i are positive for $i = 1, 2$, and $\beta_1\beta_2 - \gamma^2 > 0$, and $\alpha_i\beta_j - \alpha_j\gamma > 0$ for $i \neq j$. Similar derivation can be applied to general cases of multiple firms (see e.g. Vives 1999; Häckner 2000).

The FOC is

$$\alpha_i - 2\beta_i q_i(D) - \gamma q_k(D) - c_i(D) = 0.$$

Solving the system, we have

$$q_i(D) = \frac{(2\alpha_i\beta_k - \alpha_k\gamma) - (2\beta_k c_i(D) - \gamma c_k(D))}{4\beta_i\beta_k - \gamma^2} \text{ and } p_i(D) = \beta_i q_i + c_i(D)$$

Hence

$$\pi_i(D) = [p_i(D) - c_i(D)]\, q_i(D) = \beta_i q_i^2(D)$$

For technology 1, we have

$$\begin{aligned} s_1(R, u_i) =& \frac{1}{2}(u_i(\{r_1\}) - u_i(\emptyset)) + \frac{1}{2}(u_i(R) - u_i(\{r_2\})) \\ =& \frac{((2-\beta_k^2)\theta_i - \gamma\theta_k)\left(\epsilon_{r_1}\left(q_i(\{r_1\}) + q_i(\emptyset)\right) + (\epsilon_R - \epsilon_{r_2})\left(q_1(R) + q_1(\{r_2\})\right)\right)}{2(4\beta_i\beta_k - \gamma^2)}. \end{aligned}$$

Thus, for technology r_j, we have

$$\begin{aligned} & y_i(u, r_j) \\ =& \left(1 + \frac{(\theta_k(2-\beta_i^2) - \gamma\theta_i)\left(\epsilon_{r_j}\left(q_k(\{r_j\}) + q_k(\emptyset)\right) + \left(\epsilon_R - \epsilon_{R\backslash\{r_j\}}\right)\left(q_k(R) + q_k(R\backslash\{r_j\})\right)\right)}{(\theta_i(2-\beta_k^2) - \gamma\theta_k)\left(\epsilon_{r_j}\left(q_i(\{r_j\}) + q_i(\emptyset)\right) + \left(\epsilon_R - \epsilon_{R\backslash\{r_j\}}\right)\left(q_i(R) + q_i(R\backslash\{r_j\})\right)\right)}\right)^{-1} \end{aligned}$$

Consider prefect compatible case such that $\epsilon_{12} = \epsilon_1 + \epsilon_2$. Then we have

$$y_i(u, r_j) = \left(1 + \frac{(\theta_k(2-\beta_i^2) - \gamma\theta_i)\sum_{D\subseteq R} q_k(D)}{(\theta_i(2-\beta_k^2) - \gamma\theta_k)\sum_{D\subseteq R} q_i(D)}\right)^{-1}. \square$$

Part (b-iii) naturally follows from Proposition 1: if firms are completely symmetric, their liabilities are equal. Part (b-ii) is similar to Example 3: when markets are symmetric, the share depends explicitly on the relative productive efficiencies. Note that when the market linkage γ is low, the ratio mainly depends on the relative efficiency θ_i/θ_j. Part (b-i) shows that when firms face different markets with the same productive efficiencies, our FRAND royalties would be different from the Top-Down approach and the Proportional approach. Part (a) shows that the liability ratio depends on both productive efficiencies and market asymmetries jointly.

B.1.2 Vertical Market

For simplicity, consider only one upstream firm and $n-1$ downstream firms. Let the upstream firm to be firm 1. Under quadratic utility of the representative consumer, the demand function for firm $i = 2, \ldots, n$ is $p_i = \alpha - \beta q_i - \gamma \sum_{j\neq i,1} q_j$. When firms have access to the set of technologies $D \subseteq R$, they have a constant marginal cost c_D. The profit of the upstream firm $\pi_1 = dQ = d\sum_{j\in N\backslash\{1\}} q_j$ and the profit of a downstream firm $i \in N\backslash\{1\}$ is $\pi_i = (p_i - c_D - d)q_i = (\alpha - \beta q_i - \gamma\sum_{j\neq 1,i} q_j - c_D - d)q_i$. Consider a two-stage game where the upstream firm first decides the prices of intermediate inputs, and then all downstream firms engage in a heterogeneous Cournot competition.

Proposition 5 *Suppose the upstream firm decides the prices of intermediate inputs and downstream firms engage in a heterogeneous Cournot competition. The liability for technology $r_j \in R$ for the upstream firm is*

$$y_1(u, r_j) = \frac{2\beta + n\gamma}{3\beta + n\gamma},$$

and the liability of downstream firms $i = 2, ..., N$ are

$$y_i(u, r_j) = \frac{\beta}{(n-1)(3\beta + n\gamma)}.$$

Proof. Given the inverse demand for downstream firms $i = 2, ..., n$ is $p_i = \alpha - \beta q_i - \gamma \sum_{h \neq 1,i} q_h$, the profit maximization problem for firm i is given by

$$\max_{q_i} \pi_i(D) = \left(\alpha - \beta q_i - \gamma \sum_{h \neq 1,i} q_h - c_D - d \right) q_i.$$

The FOC for firm i is $\alpha - 2\beta q_i - \gamma \sum_{h \neq i,1} q_h - c_D - d = 0$ so that solving $n-1$ FOCs, we have

$$q_i(D) = \frac{\alpha - (c_D + d)}{2\beta + n\gamma} \text{ and } u_i(D) = \beta q_i^2(D) \text{ for all } i \in L.$$

By backward induction, the profit maximization problem for the upstream firm 1 is given by

$$\max_{d} \pi_1(D) = d \sum_{i \in L} q_i(D).$$

The FOC is $\alpha - (c_D + 2d) = 0$ so that we have

$$d = \frac{\alpha - c_D}{2} \text{ and } u_1(D) = \frac{(\alpha - c_D)(n-1)}{4(2\beta + n\gamma)}.$$

Therefore, we have

$$\pi_i(D) = \frac{\beta(\alpha - c_D)^2}{4(2\beta + n\gamma)^2} \text{ for all } i \in L.$$

For downstream firms $i \in L$, Shapley value for (R, u_i) with respect to technology r_j is

$$\begin{aligned}
s_j(R, u_i) &= \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n - |D| - 1)!}{n!} (u_i(c_{D \cup \{r_j\}}) - u_i(c_D)) \\
&= \frac{\beta(\alpha - c_D)^2}{4(2\beta + n\gamma)^2} \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n - |D| - 1)!}{n!} \left[(\alpha - c_{D \cup \{r_j\}})^2 - (\alpha - c_D)^2 \right].
\end{aligned}$$

and for the upstream firm, the Shapley value for (R, u_1)

$$\begin{aligned} s_j(R,u_1) &= \sum_{D\subseteq R\backslash\{r_j\}} \frac{|D|!(n-|D|-1)!}{n!}(u_1(c_{D\cup\{r_j\}})-u_1(c_D)) \\ &= \frac{(n-1)(\alpha-c_D)^2}{4(2\beta+n\gamma)} \sum_{D\subseteq R\backslash\{r_j\}} \frac{|D|!(n-|D|-1)!}{n!}\left[(\alpha-c_{D\cup\{r_j\}})^2-(\alpha-c_D)^2\right]. \end{aligned}$$

Hence, for each technology $r_j \in R$, we have for each downstream firm $i \in L$

$$\begin{aligned} y_i(u,r_j) &= \frac{s_j(R,u_i)}{\sum_{h\in L} s_j(R,u_h)+s_j(R,u_1)} \\ &= \frac{\frac{\beta(\alpha-c_D)^2}{4(2\beta+n\gamma)^2}}{(n-1)\frac{\beta(\alpha-c_D)^2}{4(2\beta+n\gamma)^2}+\frac{(n-1)(\alpha-c_D)^2}{4(2\beta+n\gamma)}} = \frac{\beta}{(n-1)(3\beta+n\gamma)}, \end{aligned}$$

and for the upstream firm 1, we have

$$\begin{aligned} y_1(u,r_j) &= \frac{s_j(R,u_1)}{\sum_{h\in L} s_j(R,u_h)+s_j(R,u_1)} \\ &= \frac{\frac{(n-1)(\alpha-c_D)^2}{4(2\beta+n\gamma)}}{(n-1)\frac{\beta(\alpha-c_D)^2}{4(2\beta+n\gamma)^2}+\frac{(n-1)(\alpha-c_D)^2}{4(2\beta+n\gamma)}} = \frac{2\beta+n\gamma}{3\beta+n\gamma}.\square \end{aligned}$$

Note that the level of substitutability of products of different firms, γ, enters the liability. We can check that $\frac{dy_1(u,r_j)}{d\gamma} > 0$, indicating that the upstream firm will pay more royalty as the downstream firms become more competitive; and $\frac{dy_i(u,r_j)}{d\gamma} < 0$ indicating that downstream firms pay less royalty as competition increases.

In line with the result from the single product market case, as the number of downstream firms increases, the liability of the upstream firm increases while the liabilities for all downstream firms decrease.

As market demand becomes more elastic, the same percentage of cost saving will lead to smaller improvement in the profit and thereby smaller liability for the upstream firm (i.e., $\frac{dy_1}{d\beta} < 0$).

Bertrand Market

We shows that Bertrand model deliver the similar results as Cournot model.

Horizontal Market

First, it is easy to see that Proposition 1 remains true when firms are symmetric. Hence, we focus on the asymmetric case. Since the analyses under single product market and multiple product markets are similar under Bertrand competition, we consider the multiple product case and show results similar to Propositions 2 and 3.

Consider two downstream firms (indexed by 1 and 2). The demand function for firm $i = 1, 2$ is $q_i = \alpha_i - \beta_i p_i + \gamma p_j$.Let $c_1(D) = c - \theta_1 \epsilon_D$ and $c_2(D) = c - \theta_2 \epsilon_D$ be the marginal costs for firms 1 and 2 respectively.

Proposition 6. Consider a horizontal cooperative agreement between 2 firms engaging in Bertrand competition, producing heterogenous products. Suppose technologies are perfectly compatible ($\epsilon_R = \epsilon_{r_1} + \epsilon_{r_2}$). Under FRAND compensation, the liability for $r_j \in \{r_1, r_2\}$ by firm $i \in \{1, 2\}$ is

$$y_i(u, r_j) = \left(1 + \frac{(\theta_k(2\beta_i\beta_k - \gamma^2) - \theta_i\gamma\beta_i) \sum_{D\subseteq R} q_k(D)}{(\theta_i(2\beta_i\beta_k - \gamma^2) - \theta_k\gamma\beta_k) \sum_{D\subseteq R} q_i(D)}\right)^{-1} \text{ where } k = \{1, 2\} \setminus \{i\}.$$

Proof. Profit maximization problem of firm i is given by

$$\max_{p_i} \pi_i(D) = (p_i - c_i(D)) q_i = (p_i - c_i(D)) (\alpha_i - \beta_i p_i + \gamma p_k).$$

for $i \neq k$, and $i, k = 1$ or 2. Then FOCs implies that

$$\alpha_i - \beta_i p_i + \gamma p_k - \beta_i (p_i - c_i(D)) = 0.$$

Solving the FOCs for both firms, we have

$$p_i(D) = \frac{2\alpha_i\beta_j + \gamma\alpha_j + 2\beta_i\beta_j c_i(D) + \gamma\beta_j c_j(D)}{4\beta_i\beta_j - \gamma^2},$$

and

$$\begin{aligned} q_i(D) &= \alpha_i - \beta_i p_i(D) + \gamma p_k(D) = \beta_i (p_i - c_i(D)) \\ &= \beta_i \frac{2\alpha_i\beta_k + \gamma\alpha_k - (2\beta_i\beta_k - \gamma^2) c_i(D) + \gamma\beta_k c_k(D)}{4\beta_i\beta_k - \gamma^2}. \end{aligned}$$

Hence

$$\pi_i(D) = (p_i(D) - c_i(D)) q_i = \frac{1}{\beta_i} q_i^2(D).$$

Note that

$$\begin{aligned} u_i(D \cup \{r_j\}) - u_i(D) &= \frac{1}{\beta_i} \left(q_i^2(D \cup \{r_j\}) - q_i^2(D)\right) \\ &= \frac{(2\beta_i\beta_k - \gamma^2)\theta_i - \gamma\beta_k\theta_j}{4\beta_i\beta_k - \gamma^2} \left(\epsilon_{D\cup\{r_j\}} - \epsilon_D\right) \left(q_i^2(D \cup \{r_j\}) + q_i^2(D)\right). \end{aligned}$$

Now the Shapely share by firm i for technology 1, we have

$$
\begin{aligned}
& s_j(R, u_i) \\
& = \frac{1}{2}(u_i(\{r_j\}) - u_i(\emptyset)) + \frac{1}{2}(u_i(R) - u_i(\{r_k\})) \\
& = \frac{1}{2\left(4\beta_i\beta_k - \gamma^2\right)} \left(\left(\theta_i\left(2\beta_i\beta_k - \gamma^2\right) - \theta_k\gamma\beta_k\right)\left(\epsilon_{r_j}\left(q_i\left(\{r_j\}\right) + q_i\left(\emptyset\right)\right) + \left(\epsilon_R - \epsilon_{r_k}\right)\left(q_i\left(R\right) + q_i\left(\{r_k\}\right)\right)\right)\right)
\end{aligned}
$$

where $k \neq j$.

Thus, for technology r_j, we have

$$
\begin{aligned}
& y_i(u, r_j) \\
& = \left(1 + \frac{\left(\theta_k(2\beta_i\beta_k - \gamma^2) - \theta_i\gamma\beta_i\right)\left(\epsilon_{r_j}\left(q_k\left(r_j\right) + q_k\left(\phi\right)\right) + \left(\epsilon_R - \epsilon_{R\backslash\{r_j\}}\right)\left(q_k\left(R\right) + q_k\left(R\backslash\{r_j\}\right)\right)\right)}{\left(\theta_i\left(2\beta_i\beta_k - \gamma^2\right) - \theta_k\gamma\beta_j\right)\left(\epsilon_{r_j}\left(q_i\left(r_j\right) + q_i\left(\phi\right)\right) + \left(\epsilon_R - \epsilon_{R\backslash\{r_j\}}\right)\left(q_i\left(R\right) + q_i\left(R\backslash\{r_j\}\right)\right)\right)}\right)^{-1}.
\end{aligned}
$$

Consider prefect compatible case such that $\epsilon_{12} = \epsilon_1 + \epsilon_2$. Then we have

$$
y_i(u, r_j) = \left(1 + \frac{\left(\theta_k(2\beta_i\beta_k - \gamma^2) - \theta_i\gamma\beta_i\right)\sum_{D\subseteq R} q_k\left(D\right)}{\left(\theta_i\left(2\beta_i\beta_k - \gamma^2\right) - \theta_k\gamma\beta_k\right)\sum_{D\subseteq R} q_i\left(D\right)}\right)^{-1}.\square
$$

Vertical Market

First, it is easy to see that Proposition 4 remains true under single product market. Second, we consider multiple product case and show that the result is similar to Proposition 5 .consider 1 upstream firm and $n-1$ downstream firms. Denote L the set of downstream firms. The demand function for firm $i = 2, ..., n$ is

$$
q_i = \alpha - \beta p_i + \gamma \sum_{k\in L\backslash\{i\}} p_k.
$$

Proposition 7. Consider a vertical cooperative agreement between 1 upstream firm, and $n-1$ downstream firms. Downstream firms engaging in Bertrand competition, producing heterogenous products. Suppose technologies are perfectly compatible ($\epsilon_R = \epsilon_{r_1} + \epsilon_{r_2}$). Under FRAND compensation, the liability for $r_j \in R$ for the upstream firm is

$$
y_1\left(u, r_j\right) = \frac{2\beta - (n-2)\gamma}{3\beta - 2(n-2)\gamma},
$$

and the liabilities for the downstream firm $i = 2, ..., n$ are

$$
y_i\left(u, r_j\right) = \frac{\beta - (n-2)\gamma}{(n-1)(3\beta - 2(n-2)\gamma)}.
$$

Proof. Profit maximization problem for downstream firm $i \in L$ is given by

$$
\max_{p_i} \pi_i\left(D\right) = (p_i - c_D - d)(\alpha - \beta p_i + \gamma \sum_{k\in L\backslash\{i\}} p_k),
$$

FOCs for downstream firm $i \in L$ are

$$\alpha - \beta p_i + \gamma \sum_{j \neq i,1} p_j - \beta \left(p_i - c_D - d\right) = 0.$$

Solving all $n-1$ FOCs, we have

$$p_i(D) = \frac{\alpha + \beta \left(c_D + d\right)}{2\beta - \left(n-2\right)\gamma}.$$

Hence we have

$$\begin{aligned} q_i(D) &= \alpha - \beta p_i(D) + \gamma \sum_{j \neq i} p_j = \beta \left(p_i(D) - c_D - d\right) \\ &= \beta \left(\frac{\alpha - \left(\beta - \gamma \left(n-2\right)\right)\left(c_D + d\right)}{2\beta - \left(n-2\right)\gamma} \right). \end{aligned}$$

and

$$\pi_i(D) = (p_i - c_D - d) q_i = \frac{1}{\beta} q_i^2.$$

By backward induction, the upstream firm have the following objective function

$$\max_d \pi_1\left(D\right) = d \sum_{i \in L} q_i = d\left(n-1\right)\beta \left(\frac{\alpha - \left(\beta - \gamma\left(n-2\right)\right)\left(c_D + d\right)}{2\beta - \left(n-2\right)\gamma} \right).$$

We have FOC that $\alpha - \left(\beta - \gamma\left(n-2\right)\right)\left(c_D + 2d\right) = 0$ so that

$$d = \frac{\alpha - c_D\left(\beta - \left(n-2\right)\gamma\right)}{2\left(\beta - \left(n-2\right)\gamma\right)}.$$

Therefore, we have

$$\begin{aligned} \pi_1\left(D\right) &= \frac{n-1}{4\left(2\beta - \left(n-2\right)\gamma\right)\left(\beta - \left(n-2\right)\gamma\right)} \beta\left(\alpha - c_D\left(\beta - \left(n-2\right)\gamma\right)\right), \text{ and} \\ \pi_i\left(D\right) &= \frac{\beta\left(\alpha - c_D\left(\beta - \left(n-2\right)\gamma\right)\right)^2}{4\left(2\beta - \left(n-2\right)\gamma\right)^2} \text{ for all } i \in L. \end{aligned}$$

For downstream firms $i \in L$, recall that Shapely value for (R, u_i) with respect to technology r_j is

$$\begin{aligned} s_j(R, u_i) &= \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n-|D|-1)!}{n!} \left(u_i(c_{D \cup \{r_j\}}) - u_i(c_D)\right) \\ &= \frac{\beta}{4\left(2\beta - \left(n-2\right)\gamma\right)^2} \\ &\times \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n-|D|-1)!}{n!} \left[\left(\alpha - \left(\beta - \left(n-2\right)\gamma\right) c_{D \cup \{r\}}\right)^2 - \left(\alpha - \left(\beta - \left(n-2\right)\gamma\right) c_D\right)^2 \right]. \end{aligned}$$

and for the upstream firm, the Shapely value for (R, u_1) with respect to technology r_j is

$$\begin{aligned}
s_j(R, u_1) &= \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n-|D|-1)!}{n!} (u_1(c_{D \cup \{r_j\}}) - u_1(c_D)) \\
&= \frac{n-1}{4\left(2\beta - (n-2)\gamma\right)\left(\beta - (n-2)\gamma\right)} \beta \\
&\times \sum_{D \subseteq R \setminus \{r_j\}} \frac{|D|!(n-|D|-1)!}{n!} \left[\left(\alpha - \left(\beta - (n-2)\gamma\right) c_{D \cup \{r_j\}}\right)^2 - \left(\alpha - \left(\beta - (n-2)\gamma\right) c_D\right)^2 \right].
\end{aligned}$$

Hence, the FRAND share for downstream firms $i \in L$ is

$$y_i(u, r) = \frac{s(R, u_i)}{\sum_{h \in L} s(R, u_h) + s(R, u_1)} = \frac{\beta - (n-2)\gamma}{(n-1)\left[3\beta - 2(n-2)\gamma\right]},$$

and that for the upstream firm is

$$y_1(u, r) = \frac{s(R, u_1)}{\sum_{h \in L} s(R, u_h) + s(R, u_1)} = \frac{2\beta - (n-2)\gamma}{3\beta - 2(n-2)\gamma}. \square$$